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Generalized heavy baryon chiral perturbation theory

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Abstract. Standard $SU(2)$ Heavy Baryon Chiral Perturbation Theory is extended in order to include the case of small or even vanishing quark condensate. The effective lagrangian is given to $\mathcal{O}(p^2)$ in its most general form and to $\mathcal{O}(p^3)$ in the scalar sector. A method is developed to efficiently construct the relativistic baryonic effective lagrangian for chiral SU(2) to all orders in the chiral expansion. As a first application, mass- and wave-function renormalization as well as the scalar form factor of the nucleon is calculated to $\mathcal{O}(p^3)$. The result is compared to a dispersive analysis of the nucleon scalar form factor adopted to the case of a small quark condensate. In this latter analysis, the shift of the scalar form factor between the Cheng-Dashen point and zero momentum transfer is found to be enhanced over the result assuming strong quark condensation by up to a factor of two, with substantial deviations starting to be visible for $r = m_s/\hat{m} \lesssim 12$.

1 Introduction

It is generally accepted that the chiral symmetry of massless QCD is realized in the Nambu-Goldstone mode. More precisely, it is ascertained that the QCD vacuum spontaneously breaks chiral symmetry down to the diagonal subgroup $U(3)_V$. This can actually been proven from first principles if a vanishing θ-vacuum is assumed, and provided there are $N_f \geq 3$ massless flavours and no coloured states in the spectrum (colour confinement). [1,2] In accordance with Goldstone's theorem, eight massless Goldstone bosons appear, each coupled via the coupling constant F_0 to a conserved axial-vector current. The physics of these Goldstone bosons can be described by a low energy effective theory, called Chiral Perturbation Theory (ChPT) [3,4]. The masses of the Goldstone bosons are generated by explicit symmetry breaking terms in proportion to m_q, the masses of the light quarks $q = u, d$, and s. Since m_q is small compared to the typical mass scale $\Lambda_H \approx 1$ GeV of the lightest massive hadrons not protected by chiral symmetry, the effects of m_q can be treated as a perturbation.

The coupling constant F_0 is an order parameter and a non-vanishing value $F_0 \neq 0$ is a necessary and sufficient condition for spontaneous breakdown of chiral symmetry (SBχS). The actual mechanism of SBχS is not yet understood, however. The light quark condensate in the chiral limit, $\langle\bar{q}q\rangle$, and the dimensionful parameter

$$B_0 = -F^{-2}\,\langle\bar{q}q\rangle \tag{1.1}$$

play a special role in this respect. Two scenarios seem to be theoretically viable: Large B_0 in the range of the mass scale Λ_H, corresponding to strong condensation of quarks in the QCD vacuum, or small B_0 in the range of F_0 (or even zero) corresponding to SBχS realized via extended delocalized quarks with high "mobility". [5] Although Lattice QCD simulations seem to point towards a large quark condensate, there are other attempts like in [6] where a small condensate is reported. Given the uncertainties inherent in these methods, it is fair to assume the problem to be theoretically undecided. For a critical discussion of the evidence resulting from QCD sum rules, we refer to the review article by Stern [7].

In the standard formulation of ChPT a large quark condensate, say $B_0 \approx \Lambda_H$ is assumed. The aim of generalized ChPT (GChPT) is to relax this assumption, *i.e.* to allow for small or even vanishing B_0. [8] Technically speaking, these assumptions amount to different counting rules for the light quark masses and the quark condensate, *i.e.* [8,9]

$$m_q = \mathcal{O}(p^2), \quad B_0 = \mathcal{O}(1) \qquad \text{standard ChPT} \tag{1.2}$$
$$m_q = \mathcal{O}(p), \quad B_0 = \mathcal{O}(p) \qquad \text{generalized ChPT}, \tag{1.3}$$

where p is a generic symbol for a low energy quantity. GChPT thus reorders the expansion of the effective lagrangian of low energy QCD. Summed up to all orders it coincides with the standard approach. At each finite order, however, GChPT takes into account contributions which in the standard case are treated as higher order terms. Since this reordering concerns the explicit symmetry breaking sector only, the difference between standard and generalized ChPT will be in proportion to the light

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quark masses and hence small[1]. The generalized approach allows to experimentally probe the size of the quark condensate [9]. The most promising case to discriminate experimentally between the two scenarios is provided by low-energy $\pi\pi$-scattering, [11] where precise data should be available in the near future. [12]

Chiral symmetry also restricts the low energy interactions of pions with baryons. Using the so called heavy baryon formalism (HBChPT), the πN–system at low energies has been investigated extensively and put to many tests. [13] It is natural to ask whether it is possible to gain insight into the mechanism of SBχS from the baryonic sector, as well. Incidentally, the first evidence for a possible small light quark condensate was obtained by an analysis of the Goldberger-Treiman discrepancy. [14] However, this quantity turned out to be very sensitive to the pion-nucleon coupling constant and therefore the analysis remained inconclusive. Also, only the leading order corrections were considered. Other single baryon processes like $\pi N \to N\pi$ or $\gamma N \to N\pi\pi$, where abundant and precise data are available, are potential candidates for testing the assumption of large B_0 made in SChPT. In order to make such tests possible, however, HBChPT has to be adapted to the principles of the generalized approach. Any calculation performed in SHBChPT can provide consistence checks only, of course. The aim of the present article is to fill this gap, *i.e.* to formulate generalized heavy baryon chiral perturbation theory (GHBChPT). Two crucial assumptions will be made. First, the quark mass counting rules of GChPT as given in (1.3) will be employed. This follows directly from the pure Goldstone Boson sector. Second, we assume that the expectation values of non-singlet operators between one-nucleon states scale with Λ_χ^D, where $\Lambda_\chi \approx 1$ GeV and D denotes the canonical mass dimensions of these correlation functions (no other small scales like B_0 present). In particular, we treat dimensionless couplings like e_1 in the effective πN–lagrangian

$$\mathcal{L}_{\pi N} = e_1 \bar{\Psi} \mathrm{tr}(\chi^\dagger U + U^\dagger \chi)\Psi, \quad \chi = s + ip, \tag{1.4}$$

as quantity of order unity. Here, Ψ denotes the nucleon field, s and p are scalar and pseudoscalar sources, respectively, and U contains the pion field in the usual manner (*c.f.* Sect. 2 for definitions). The term in (1.4) counts therefore as order p.

Having made these assumptions, GHBChPT can be formulated along the same lines as in the standard case. At each order in the chiral expansion, the effective lagrangian contains additional terms compared to the standard formulation. One of the main problems will be to obtain estimates for these additional coupling constants. Given the many observables available in the πN–system, the task is not hopeless. It remains to be seen whether similar clean tests as those in $\pi\pi$–scattering can be devised, ultimately leading to a better understanding of the mechanism of spontaneous chiral symmetry breakdown. This work is the first step of such a program.

[1] Recently, interesting consequences of a vanishing light quark condensate have been derived for the spectrum of vector- and axial-vector states in the large N_c-limit of QCD. [10]

The article is organized as follows. In Sect. 2 the effective chiral lagrangian of GHBChPT is given to $\mathcal{O}(p^2)$ in it's most general form and to $\mathcal{O}(p^3)$ in the scalar sector. Mass- and wave-function renormalization to order p^3 are considered in Sect. 3. In Sect. 4 we calculate the scalar form factor of the nucleon to one-loop and compare with a dispersive theoretic evaluation adapted to the case of a small quark condensate. Finally, we draw the conclusions in Sect. 5. In Appendix A we give a method to efficiently construct the relativistic baryonic effective lagrangian for chiral SU(2) to all orders. Appendix B contains a collection of loop functions employed in the article.

2 The effective heavy baryon lagrangian in the generalized approach

Our starting point is the QCD lagrangian with two massless quarks coupled to external sources [4][2]

$$\mathcal{L} = \mathcal{L}^0_{\mathrm{QCD}} + \bar{q}\gamma^\mu \left(v_\mu + \frac{1}{3} v^{(s)}_\mu + \gamma_5 a_\mu \right) q - \bar{q}(s - i\gamma_5 p) q,$$
$$q = \begin{pmatrix} u \\ d \end{pmatrix}, \tag{2.1}$$

where $\mathcal{L}^0_{\mathrm{QCD}}$ is the lagrangian of two-flavour QCD in the absence of external sources. The isotriplet vector (axial-vector) currents v_μ (a_μ) are hermitian and traceless matrix fields, whereas the current $v^{(s)}_\mu$ is an isosinglet. The scalar and the pseudo-scalar sources, s and p respectively, are also hermitian matrix fields in isospin space, but in general are not traceless. The lagrangian (2.1) is symmetric with respect to chiral transformations $g \in G = SU(2)_L \times SU(2)_R$. The QCD vacuum is assumed to spontaneously break chiral symmetry down to the diagonal subgroup $H = SU(2)_V$. According to Goldstone's theorem, the spectrum of the theory contains three massless states ϕ, the pions, which are gathered in $u(\phi)$, an element of the chiral coset space, *i.e.* $u(\phi) \in G/H$. Explicit symmetry breaking terms in proportion to the light quark masses will give small masses to the pions. Technically, this is incorporated by setting the scalar source to $s = \mathcal{M} = \mathrm{diag}\,(m_u, m_d)$ at the end of the calculation.

The low-energy effective theory of pions and nucleons is obtained by the CCWZ construction: chiral symmetry is realized non-linearly on the Goldstone fields ϕ and matter fields Ψ (nucleons) via [15]

$$u(\phi) \xrightarrow{g} g_R u(\phi) h(g,\phi)^{-1} = h(g,\phi) u(\phi) g_L^{-1}, \tag{2.2}$$
$$\Psi = \begin{pmatrix} p \\ n \end{pmatrix} \xrightarrow{g} h(g,\phi)\Psi, \tag{2.3}$$
$$g = (g_L, g_R) \in SU(2)_L \times SU(2)_R .$$

The compensator field $h(g,\phi) \in SU(2)_V$ is defined by (2.2) and characterizes the non-linear realization. The effective lagrangian consists of all invariants under chiral

[2] In this paper we consider only chiral $SU(2)$. Accordingly, the $SU(3)$ constants F_0 and B_0 are replaced by their $SU(2)$ counterparts F and B, respectively

transformations which in addition are hermitian and invariant under the discrete transformations P, C, and T. In order to construct these invariants explicitly, it is convenient to define the following fields

$$\begin{aligned} u_\mu &= i\left\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger\right\}\,, \\ \Gamma_\mu &= \frac{1}{2}\left\{u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - i\ell_\mu)u^\dagger\right\}\,, \\ \chi_\pm &= u^\dagger\chi u^\dagger \pm u\chi^\dagger u\,, \qquad \chi = (s+ip)\,, \\ f_\pm^{\mu\nu} &= uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu}u\,, \\ v_{\mu\nu}^{(s)} &= \partial_\mu v_\nu^{(s)} - \partial_\nu v_\mu^{(s)}. \end{aligned} \tag{2.4}$$

Here, the right and left handed fields $r_\mu = v_\mu + a_\mu$, $\ell_\mu = v_\mu - a_\mu$ are the external gauge fields with associated non–Abelian field strengths

$$\begin{aligned} F_R^{\mu\nu} &= \partial^\mu r^\nu - \partial^\nu r^\mu - i\left[r^\mu, r^\nu\right]\,, \\ F_L^{\mu\nu} &= \partial^\mu \ell^\nu - \partial^\nu \ell^\mu - i\left[\ell^\mu, \ell^\nu\right]\,. \end{aligned} \tag{2.5}$$

Notice the missing factor $2B$ in the definition of $\chi_\pm$ in (2.4). The covariant derivative ∇_μ acts on all fields to the right of it and is formally given by

$$\begin{aligned} \nabla_\mu &= \partial_\mu + \left(\Gamma_\mu - iv_\mu^{(s)}\right)\,, \\ \overleftarrow{\nabla}_\mu &= \overleftarrow{\partial}_\mu - \left(\Gamma_\mu - iv_\mu^{(s)}\right)\,. \end{aligned} \tag{2.6}$$

We are now ready to discuss the consequences of generalized ChPT for the pion-nucleon effective lagrangian. Before launching into the discussion of generalized heavy baryon ChPT, however, let us recapitulate some facts about the Goldstone boson sector. The idea of a small or vanishing quark condensate leads to a different counting rule for quark mass terms, and consequently to a reordering of the chiral lagrangian. Consider the expansion of the squared pion mass

$$M_\pi^2 = 2B\hat{m} + 4A\hat{m}^2\,, \quad \hat{m} = \frac{1}{2}\left(m_u + m_d\right)\,. \tag{2.7}$$

The constant A is an order parameter which has been estimated by chiral sum rules to be of the order unity [9]. In the standard approach B is assumed to be in the range set by the hadronic mass scale Λ_H. Thus the first term in (2.7) is the leading contribution and the second term is suppressed by a factor m_q. Equation (2.7)then implies the formal counting rule: $B \sim \mathcal{O}(\Lambda_H)$ and $\hat{m} \sim \mathcal{O}(\frac{p^2}{\Lambda_H})$, where p is a small external momentum. Accordingly, the effective lagrangian is ordered in local, chiral invariant terms $\mathcal{L}_{(k,l)}$, with k powers of covariant derivatives and l powers of quark mass insertions [9]

$$\mathcal{L}_{eff} = \tilde{\mathcal{L}}^{(2)} + \tilde{\mathcal{L}}^{(4)} + \dots\,, \tag{2.8}$$

where

$$\tilde{\mathcal{L}}^{(d)} = \sum_{k+2l=d} \mathcal{L}_{(k,l)}\,. \tag{2.9}$$

The tilde on $\tilde{\mathcal{L}}^{(d)}$ signals that the associated coupling constants are those of standard ChPT.

If B is small, however, both terms in (2.7) are of the same order and, in general, equally important. The quark mass m_q and the coupling constant B, thus, must count as order $\mathcal{O}(p)$. The effective lagrangian $\mathcal{L}_{eff}$ must therefore be expanded not only in powers of covariant derivatives and quark mass insertions, but in powers of B as well [8,9]

$$\mathcal{L}_{eff} = \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \dots\,, \tag{2.10}$$

where

$$\mathcal{L}^{(d)} = \sum_{j+k+l=d} B^j \mathcal{L}_{(k,l)}\,. \tag{2.11}$$

For instance, the leading order p^2 lagrangian of the Goldstone boson sector consists of terms with either two covariant derivatives and no mass insertions or of terms with no covariant derivatives and two mass insertions [16]

$$\begin{aligned} \mathcal{L}_{\pi\pi}^{(2)} = \frac{F^2}{4}\Big[&\left\langle D_\mu U D^\mu U^\dagger\right\rangle + 2B\left\langle \chi^\dagger U + \chi U^\dagger\right\rangle \\ &\times A\left\langle \left(\chi^\dagger U\right)^2 + \left(\chi U^\dagger\right)^2\right\rangle + Z_p\left\langle \chi^\dagger U - U^\dagger\chi\right\rangle^2 \\ &\times h_0\left\langle \chi^\dagger\chi\right\rangle + h_0'\left(\det\chi^\dagger + \det\chi\right)\Big]\,, \end{aligned} \tag{2.12}$$

where $U = u^2$ and the covariant derivative is defined as

$$D_\mu U = \partial_\mu U - ir_\mu U + iU\,l_\mu\,. \tag{2.13}$$

Of course, (2.12) could be expressed in terms of the chiral fields defined in (2.4).

To summarize, (2.8) and (2.10) state that summed to all orders the standard and generalized approach coincide, namely they describe the same effective lagrangian. To any finite order, however, they differ, since in the generalized case terms are taken into account which the standard case relegates to higher order.

Turning now to the baryonic sector, our starting point is the generating functional for Green functions of single nucleon processes defined by

$$\begin{aligned} e^{iZ[j,\eta,\bar{\eta}]} = \mathcal{N}\int \left[du d\Psi d\bar{\Psi}\right] \\ \times \exp\left[i\left\{\tilde{S}_M + S_{MB} + \int d^4x\left(\bar{\eta}\Psi + \bar{\Psi}\eta\right)\right\}\right]\,. \end{aligned} \tag{2.14}$$

$\tilde{S}_M$ and S_{MB} are the mesonic and relativistic pion-nucleon effective actions, respectively. The tilde on $\tilde{S}_M$ accounts for the fact that in (2.14) the nucleon degrees of freedom have not yet been integrated out. The form of the mesonic action remains the same, *c.f.* the leading order expression given in (2.12) above. The coupling constants in general are different, however, since they might get contributions from closed nucleon loops. [17] In the pion-nucleon sector the relativistic lowest-order chiral lagrangian takes the form [18]

$$\mathcal{L}^{(1)}_{\pi N} = \bar\Psi\Big[i \not\nabla - m + \frac{\overset{\circ}{g}_A}{2} \not u\gamma_5 + e_1 \langle\chi_+\rangle + e_2\left(\chi_+ - \frac{1}{2}\langle\chi_+\rangle\right) + e_3\gamma_5\langle\chi_-\rangle + e_4\gamma_5\chi_-\Big]\Psi\,. \tag{2.15}$$

A characteristic feature of generalized ChPT is the appearance of terms in proportion to $e_1,\dots e_4$ already at leading order. In standard ChPT they are present only at next-to-leading order. We have chosen new symbols for the corresponding coupling constants in order not to confuse these couplings with those of standard ChPT.

Next we take the non-relativistic limit in order to get rid of the large nucleon mass term, thereby allowing for a consistent low energy expansion. [19–21] Projecting the nucleon field Ψ onto its velocity dependent light and heavy components N_v and H_v respectively [22]

$$\begin{aligned} N_v(x) &= \exp[imv\cdot x]P_v^+\Psi(x)\,,\\ H_v(x) &= \exp[imv\cdot x]P_v^-\Psi(x)\,,\\ P_v^\pm &= \frac{1}{2}(1\pm \not v)\,, \qquad v^2 = 1\,, \end{aligned} \tag{2.16}$$

the pion–nucleon Lagrangian is brought to the form

$$\mathcal{L}_{\pi N} = \bar N_v A N_v + \bar H_v B N_v + \bar N_v\gamma^0 B^\dagger\gamma^0 H_v - \bar H_v C H_v\,. \tag{2.17}$$

The mesonic field operators A, B, C admit chiral expansions of the form

$$\begin{aligned} A &= A_{(1)} + A_{(2)} + A_{(3)} + \dots\,,\\ B &= B_{(1)} + B_{(2)} + \dots\,, \end{aligned} \tag{2.18}$$

and

$$C = C_{(0)} + C_{(1)} + \dots\,, \tag{2.19}$$

where $A_{(n)}$ denotes a quantity of order p^n. Explicitly, the leading order expressions are given by

$$\begin{aligned} A_{(1)} &= iv\cdot\nabla + \overset{\circ}{g}_A S\cdot u + e_1\langle\chi_+\rangle + e_2\left(\chi_+ - \frac{1}{2}\langle\chi_+\rangle\right)\,,\\ B_{(1)} &= i\not\nabla^\perp - \frac{\overset{\circ}{g}_A}{2} v\cdot u\gamma_5 + e_3\gamma_5\langle\chi_-\rangle + e_4\gamma_5\chi_-\,,\\ C_{(0)} &= 2m\,,\\ C_{(1)} &= iv\cdot\nabla + \overset{\circ}{g}_A\, S\cdot u - e_1\langle\chi_+\rangle - e_2\left(\chi_+ - \frac{1}{2}\langle\chi_+\rangle\right)\,,\\ \nabla^\perp_\mu &= \nabla_\mu - v_\mu v\cdot\nabla\,, \qquad S^\mu = i\gamma_5\sigma^{\mu\nu}v_\nu/2\,. \end{aligned} \tag{2.20}$$

Performing a Gaussian integration in the generating functional, the fields H_v are integrated out. Introducing sources corresponding to N_v and H_v, respectively,

$$\rho_v = e^{imv\cdot x}P_v^+\eta, \qquad R_v = e^{imv\cdot x}P_v^-\eta\,, \tag{2.21}$$

and shifting also the variable N_v, the generating functional can be brought to the form

$$\begin{aligned} e^{iZ[j,\eta,\bar\eta]} = \mathcal{N}\int [dudN_v d\bar N_v]\exp\Big[i\tilde S_M + i\int d^4x \\ \times\Big\{\bar N_v(A+\gamma^0B^\dagger\gamma^0C^{-1}B)N_v + \bar RC^{-1}R \\ -(\bar\rho + \bar RC^{-1}B)(A+\gamma^0B^\dagger\gamma^0C^{-1}B) \\ \times(\rho+\gamma^0B^\dagger\gamma^0C^{-1}R)\Big\}\Big]\,. \end{aligned} \tag{2.22}$$

In HBChPT, the matrix C^{-1} in (2.22) is expanded in a power series in $1/m$ before performing the functional integral over N_v. Then, this functional integral yields only a constant. However, as pointed out in [17], we know the effect of the interchange of limits must be to change $\tilde S_M$ into S_M, with S_M the usual effective action of ChPT. The standard procedure of ChPT can be applied, [4] and we refer to [17] for the relation of the T-matrix elements obtained in HBChPT and the fully relativistic S-matrix elements one is actually looking for. We come back to this point in Sect. 3 when we consider mass and wavefunction renormalization.

Here we are concerned only with the effective low energy action of the pion-nucleon system resulting from the analysis above

$$\begin{aligned} S_{\pi N} &= \int d^4x\ \widehat{\mathcal{L}}_{\pi N}\\ &= \int d^4x\ \bar N_v(A+\gamma^0B^\dagger\gamma^0C^{-1}B)N_v\,, \end{aligned} \tag{2.23}$$

where C^{-1} is understood to be expanded in inverse powers of the nucleon mass. We thus obtain the chiral Lagrangian of GHBCHPT up to $\mathcal{O}(p^3)$ in its general form

$$\begin{aligned} A+\gamma^0B^\dagger\gamma^0C^{-1}B &= A_{(1)}\\ &\quad + A_{(2)} + \frac{1}{2m}\gamma^0B^\dagger_{(1)}\gamma^0B_{(1)}\\ &\quad + A_{(3)} + \frac{1}{2m}\left(\gamma^0B^\dagger_{(2)}\gamma^0B_{(1)} + \gamma^0B^\dagger_{(1)}\gamma^0B_{(2)}\right)\\ &\quad - \frac{1}{4m^2}\gamma^0B^\dagger_{(1)}\gamma^0C_{(1)}B_{(1)} + \mathcal{O}(p^4)\,. \end{aligned} \tag{2.24}$$

In the following we detail the derivation of the explicit effective pion-nucleon lagrangians up to order p^3. A recipe of how to construct the relativistic invariants for chiral SU(2) to all orders in the chiral expansion is given in App. A. We follow closely the presentation given in [23].

At lowest order the lagrangian is given by $A_{(1)}$

$$\widehat{\mathcal{L}}^{(1)}_{\pi N} = \bar N_v\Big[iv\cdot\nabla + \overset{\circ}{g}_A S\cdot u + e_1\langle\chi_+\rangle + e_2\left(\chi_+ - \frac{1}{2}\langle\chi_+\rangle\right)\Big]N_v\ . \tag{2.25}$$

The operators associated with the coupling constants e_1 and e_2 are a new feature of the generalized approach. In

the standard case these two operators are treated as operators of the order $\mathcal{O}(p^2)$ and they are associated with the couplings a_3 and a_4 respectively [23].

The relativistic chiral lagrangian at order $\mathcal{O}(p^2)$ has four types of contributions:

$$\mathcal{L}^{(2)}_{\pi N} = \mathcal{L}_{(2,0)} + \mathcal{L}_{(1,1)} + \mathcal{L}_{(0,2)} + B\,\mathcal{L}^{(1)}_{\pi N}\,. \tag{2.26}$$

The term $\mathcal{L}_{(2,0)}$ is the same as in the standard case. The appearance of $\mathcal{L}_{(1,1)}$ and of $\mathcal{L}_{(0,2)}$ already at the order $\mathcal{O}(p^2)$ is characteristic of the generalized approach. $\mathcal{L}_{(1,1)}$ can be read off from the $\mathcal{O}(p^3)$ lagrangian given in [23] and $\mathcal{L}_{(0,2)}$ counted formally as an $\mathcal{O}(p^4)$ contribution. Since the loop divergences associated with virtual pion and nucleon exchange are at least of order $\mathcal{O}(p^3)$, the last term in (2.26) only leads to a finite renormalization of $\mathcal{L}^{(1)}_{\pi N}$ and may be neglected. In order to construct the effective lagrangian at $\mathcal{O}(p^2)$ we have used the following set of operators [23]:

$$\begin{aligned}
&\langle u_\mu u^\mu\rangle, \to A_{(2)}\\
&i\sigma^{\mu\nu}u_\mu u_\nu,\ \sigma^{\mu\nu}f_{+\mu\nu},\\
&\sigma^{\mu\nu}v^{(s)}_{\mu\nu},\ \langle u_\mu u_\nu\rangle i\gamma^\mu\nabla^\nu + \text{h.c.} \to A_{(2)}, B_{(2)}\\
&[\not{u}\,,\chi_-]\,,\ \gamma_5\not{u}\,\langle\chi_+\rangle\,,\gamma_5\gamma_\mu\,\langle u^\mu\chi_+\rangle\,,\\
&\gamma_5\,[\not{\nabla},\chi_-]\,,\ \gamma_5\gamma_\mu\,\langle\partial^\mu\chi_-\rangle \to A_{(2)}, B_{(2)}\\
&\langle\chi_+\rangle^2\,,\ \chi_+\,\langle\chi_+\rangle\,,\ \langle\chi_+\chi_+\rangle\,,\\
&\langle\chi_-\rangle^2\,,\ \chi_-\,\langle\chi_-\rangle\,,\ \langle\chi_-\chi_-\rangle\ \to A_{(2)}\,.
\end{aligned} \tag{2.27}$$

In the derivation of (2.27) we have omitted all operators, which can be eliminated with a redefinition of the nucleon field Ψ as given in the appendix A, e.g. the term

$$\langle u_\mu u_\nu\rangle\nabla^\mu\nabla^\nu + \text{h.c.}$$

can be eliminated with (A.43). The two pieces at order $\mathcal{O}(p^2)$ in (2.24) are then found to be

$$\begin{aligned}
A_{(2)} = {}&\frac{c_2}{m}(v\cdot u)^2 + \frac{c_3}{m}u\cdot u\\
&+\frac{1}{m}\varepsilon^{\mu\nu\rho\sigma}v_\rho S_\sigma\left[ic_4u_\mu u_\nu + c_6 f_{+\mu\nu} + c_7 v^{(s)}_{\mu\nu}\right]\\
&+\frac{c_8}{m}[\chi_-, v\cdot u] + \frac{c_9}{m}iS^\mu[\nabla_\mu,\chi_-] + \frac{c_{10}}{m}iS^\mu\langle\partial_\mu\chi_-\rangle\\
&+\frac{c_{11}}{m}S\cdot u\langle\chi_+\rangle + \frac{c_{12}}{m}S^\mu\langle u_\mu\chi_+\rangle\\
&+\frac{c_{13}}{m}\langle\chi_+\rangle^2 + \frac{c_{14}}{m}\chi_+\langle\chi_+\rangle + \frac{c_{15}}{m}\langle\chi_+\chi_+\rangle\\
&+\frac{c_{16}}{m}\langle\chi_-\rangle^2 + \frac{c_{17}}{m}\chi_-\langle\chi_-\rangle + \frac{c_{18}}{m}\langle\chi_-\chi_-\rangle\ ,
\end{aligned} \tag{2.28}$$

and

$$\begin{aligned}
&\frac{1}{2m}\gamma^0 B^\dagger_{(1)}\gamma^0 B_{(1)}\\
&\quad=\frac{1}{2m}\Big[(v\cdot\nabla)^2 - \nabla\cdot\nabla - i\mathring{g}_A\{S\cdot\nabla, v\cdot u\}\\
&\quad-\frac{\mathring{g}_A^2}{4}(v\cdot u)^2 + \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}v_\rho S_\sigma\left[iu_\mu u_\nu + f_{+\mu\nu} + 2v^{(s)}_{\mu\nu}\right]\\
&\quad-\frac{1}{2}\mathring{g}_A e_4[\chi_-, v\cdot u] + 2e_4 iS^\mu[\nabla_\mu,\chi_-] + 2e_3 iS^\mu\langle\partial_\mu\chi_-\rangle\\
&\quad+e_3^2\langle\chi_-\rangle^2 + e_4^2\chi_-\chi_- + 2e_3e_4\chi_-\langle\chi_-\rangle\Big]\,.
\end{aligned} \tag{2.29}$$

The operator $(v\nabla)^2/2m$ in (2.29) is a special case of an equation-of-motion type term. Up to $\mathcal{O}(p^3)$ we have encountered the following equation-of-motion type terms [23]:

$$\begin{aligned}
\mathcal{L}_{\text{EOM}} = \bar{N}_v\Big\{&X\,(iv\cdot\nabla)^3 + iv\cdot\overleftarrow{\nabla}Yiv\cdot\nabla\\
&+Ziv\cdot\nabla - iv\cdot\overleftarrow{\nabla}\gamma^0 Z^\dagger\gamma^0\Big\}N_v\ .
\end{aligned} \tag{2.30}$$

The operators $Y = Y^\dagger$ and Z are purely mesonic operators and are at most of the order $\mathcal{O}(p)$ and $\mathcal{O}(p^2)$ respectively and X is a real constant. Applying an adaptation of the field redefinition given in [23]

$$\begin{aligned}
N_v = \Big\{&1 - \frac{X}{2}(iv\cdot\nabla)^2 + \frac{1}{2}\left[Y + \Delta\mathcal{L}^{(1)}\right]iv\cdot\nabla\\
&+\frac{X}{2}\left[iv\cdot\nabla, \Delta\mathcal{L}^{(1)}\right]\\
&-\frac{X}{2}\left(\Delta\mathcal{L}^{(1)}\right)^2 - \frac{Y}{2}\Delta\mathcal{L}^{(1)} - \gamma^0 Z^\dagger\gamma^0\Big\}N'_v
\end{aligned} \tag{2.31}$$

with

$$\Delta\mathcal{L}^{(1)} = \mathring{g}_A S\cdot u + e_1\langle\chi_+\rangle + e_2\left(\chi_+ - \frac{1}{2}\langle\chi_+\rangle\right)$$

to the leading order lagrangian $\widehat{\mathcal{L}}^{(1)}_{\pi N}$ eliminates all equation-of-motion terms. It also induces a lagrangian at order $\mathcal{O}(p^2)$ which in our case with $X = Z = 0$ and $Y = 1/(2m)$ is explicitly given by

$$\mathcal{L}_{\text{ind}} = -\frac{1}{2m}\bar{N}'_v\left(\Delta\mathcal{L}^{(1)}\right)^2 N'_v\,. \tag{2.32}$$

Naturally this field redefinition induces a lagrangian at order $\mathcal{O}(p^3)$ which must be dealt with in the construction of $\widehat{\mathcal{L}}^{(3)}_{\pi N}$. Adding all the pieces together the effective lagrangian at order $\mathcal{O}(p^2)$ is then given in its final form by

$$\begin{aligned}
\widehat{\mathcal{L}}^{(2)}_{\pi N} = \bar{N}_v\Big[&-\frac{1}{2m}\left(\nabla\cdot\nabla + i\mathring{g}_A\{S\cdot\nabla, v\cdot u\}\right)\\
&+\frac{f_2}{m}\langle(v\cdot u)^2\rangle + \frac{f_3}{m}\langle u\cdot u\rangle\\
&+\frac{1}{m}\varepsilon^{\mu\nu\rho\sigma}v_\rho S_\sigma\left[if_4u_\mu u_\nu + f_6 f_{+\mu\nu} + f_7 v^{(s)}_{\mu\nu}\right]\\
&+\frac{f_8}{m}[\chi_-, v\cdot u] + \frac{f_9}{m}iS^\mu[\nabla_\mu,\chi_-]\\
&+\frac{f_{10}}{m}iS^\mu\langle\partial_\mu\chi_-\rangle\\
&+\frac{f_{11}}{m}S\cdot u\langle\chi_+\rangle + \frac{f_{12}}{m}S^\mu\langle u_\mu\chi_+\rangle\\
&+\frac{f_{13}}{m}\langle\chi_+\rangle^2 + \frac{f_{14}}{m}\chi_+\langle\chi_+\rangle + \frac{f_{15}}{m}\langle\chi_+{}^2\rangle\\
&+\frac{f_{16}}{m}\langle\chi_-\rangle^2 + \frac{f_{17}}{m}\chi_-\langle\chi_-\rangle + \frac{f_{18}}{m}\langle\chi_-{}^2\rangle\Big]N_v\ .
\end{aligned} \tag{2.33}$$

The coupling constants f_i are related to those appearing previously by

$$
\begin{aligned}
f_2 &= \frac{c_2}{2} - \frac{\overset{\circ}{g}_A^2}{8} & f_3 &= \frac{c_3}{2} + \frac{\overset{\circ}{g}_A^2}{16} \\
f_4 &= c_4 + \frac{1-\overset{\circ}{g}_A^2}{4} & f_6 &= c_6 + \frac{1}{4} \\
f_7 &= c_7 + \frac{1}{2} & f_8 &= c_8 - \frac{1}{4}\overset{\circ}{g}_A e_4 \\
f_9 &= c_9 + e_4 & f_{10} &= c_{10} + e_3 \\
f_{11} &= c_{11} - \overset{\circ}{g}_A e_1 & f_{12} &= c_{12} - \frac{\overset{\circ}{g}_A e_2}{2} \\
f_{13} &= c_{13} - \frac{e_1^2}{2} + \frac{e_1 e_2}{2} + \frac{e_2^2}{8} & f_{14} &= c_{14} - e_1 e_2 \\
f_{15} &= c_{15} - \frac{e_4^2}{4} & f_{16} &= c_{16} + \frac{e_3^2}{2} - \frac{e_4^2}{4} \\
f_{17} &= c_{17} + e_3 e_4 + \frac{e_4^2}{2} & f_{18} &= c_{18} + \frac{e_4^2}{4}\,.
\end{aligned}
$$

At order $\mathcal{O}(p^3)$ we restrict ourselves to operators which contribute to either mass- and wave function renormalization of the nucleon field Ψ or to the scalar sector of the πN-system. This means that we set all chiral fields to zero except for the scalar source χ_+. The non relativistic lagrangian $\widehat{\mathcal{L}}^{(3)}_{\pi N}$ has the following contributions:

1. As stated before the generalized approach is also an expansion in the parameter B. Therefore the relativistic lagrangian at order $\mathcal{O}(p^3)$ includes the two terms of the form
$$B^2\,\mathcal{L}^{(1)}_{\pi N} + B\,\mathcal{L}^{(2)}_{\pi N}\,. \tag{2.34}$$
In the non-relativistic limit their B-dependent contributions have the same chiral structure as $A_{(1)}$ and $A_{(2)}$, but have new coupling constants. These B-dependent counterterms of $\mathcal{O}(p^3)$ are needed to renormalize divergences that arise from using the vertices of $\widehat{\mathcal{L}}^{(1)}_{\pi N}$ in the loop. They renormalize the coupling constants of $\widehat{\mathcal{L}}^{(1)}_{\pi N}$ by contributions of $\mathcal{O}(B^2)$ and the constants of $\widehat{\mathcal{L}}^{(2)}_{\pi N}$ by an amount of $\mathcal{O}(B)$. As can bee seen from the lowest order lagrangian, B enters only via the product $\hat{m}B$, thus there is no loop-divergence in proportion to $B^2\hat{m}$. We can therefore omit all operators of the form $B^2\,\mathcal{L}^{(1)}_{\pi N}$.
2. The relativist lagrangian $\mathcal{L}^{(3)}_{\pi N}$ contains genuine new operators that contribute in the non-relativistic limit to $A_{(3)}$, see (2.24):
$$
\begin{aligned}
\sum_{i=1}^{7}\tilde{g}_i\mathcal{O}_i = {}& \frac{\tilde{g}_1}{m^2}\left[\nabla_\mu,[\nabla^\mu,\chi_+]\right] + \frac{\tilde{g}_2}{m^2}\left\langle\partial\cdot\partial\,\chi_+\right\rangle \\
&+\frac{\tilde{g}_3}{m^2}\chi_+\left\langle\chi_+\chi_+\right\rangle + \frac{\tilde{g}_4}{m^2}\chi_+\left\langle\chi_+\right\rangle^2 \\
&+\frac{\tilde{g}_5}{m^2}\left\langle\chi_+\right\rangle^3 + \frac{\tilde{g}_6}{m^2}\left\langle\chi_+\chi_+\right\rangle\left\langle\chi_+\right\rangle\,. \\
&+\frac{\tilde{g}_7}{m^2}\left[\chi_+,[iv\cdot\nabla,\chi_+]\right]\,.
\end{aligned}
\tag{2.35}
$$
Taking into account the terms in proportion to B we find for $A_{(3)}$
$$
\begin{aligned}
A_{(3)} = \sum_{i=1}^{7}\tilde{g}_i\mathcal{O}_i + \frac{B}{m^2}\Big[&\tilde{c}_{13}\left\langle\chi_+\right\rangle^2 \\
&+\tilde{c}_{14}\chi_+\left\langle\chi_+\right\rangle + \tilde{c}_{15}\left\langle\chi_+\chi_+\right\rangle\Big]\,.
\end{aligned}
\tag{2.36}
$$
3. The contributions from the $1/m$ expansion in (2.24). The terms proportional to $B_{(2)}$, however, do not contribute in our case.
4. The field redefinition as given in (2.31) with $X = Z = 0$ and $Y = 1/(2m)$ applied to $\widehat{\mathcal{L}}^{(1)}_{\pi N}$ and to the sum of the (2.28) and (2.29) induces a lagrangian at $\mathcal{O}(p^3)$, which must be taken into account. After this transformation there are still equation-of-motion terms at the $\mathcal{O}(p^3)$ level. These can be removed by a second transformation inducing additional terms to the $\mathcal{O}(p^3)$-lagrangian. The appropriate choice of X, Y and Z will be given below.

In a first step we collect all contributions from $1\ldots3$ and from the field transformation (2.31). Splitting the $\mathcal{O}(p^3)$-lagrangian into equation-of-motion terms and a remainder we find:
$$\widehat{\mathcal{L}}^{(3)}_{\pi N} = \widehat{\mathcal{L}}^{(3)}_{\mathrm{EOM}} + \widehat{\mathcal{L}}^{(3)}_{\mathrm{rem}} \tag{2.37}$$
with
$$
\begin{aligned}
\widehat{\mathcal{L}}^{(3)}_{\mathrm{EOM}} = \frac{1}{16m^2}\bar{N}'_v\Big\{&\bar{X}\left(iv\cdot\nabla\right)^3 + iv\cdot\overleftarrow{\nabla}\bar{Y}iv\cdot\nabla \\
&+\bar{Z}iv\cdot\nabla - iv\cdot\overleftarrow{\nabla}\gamma^0\bar{Z}^\dagger\gamma^0\Big\}N'_v\,,
\end{aligned}
\tag{2.38}
$$
where
$$
\begin{aligned}
\bar{X} &= 1\,, \\
\bar{Y} &= \Delta\mathcal{L}^{(1)}\,, \\
\bar{Z} &= \frac{1}{2}\left(\Delta\mathcal{L}^{(1)}\right)^2 + 4\Delta\mathcal{L}^{(2)} - \left[iv\cdot\overrightarrow{\nabla},\Delta\mathcal{L}^{(1)}\right]\,,
\end{aligned}
\tag{2.39}
$$
and
$$
\begin{aligned}
\widehat{\mathcal{L}}^{(3)}_{\mathrm{rem}} = \frac{1}{16m^2}\bar{N}'_v\Big\{&16m^2\sum_{i=1}^{7}\tilde{g}_i\mathcal{O}_i \\
&+\frac{1}{2}\left[\Delta\mathcal{L}^{(1)},\left[iv\cdot\nabla,\Delta\mathcal{L}^{(1)}\right]\right] + \left(\Delta\mathcal{L}^{(1)}\right)^3 \\
&-4\left\{\Delta\mathcal{L}^{(1)},\Delta\mathcal{L}^{(2)}\right\} + 2\left[\nabla_\mu,\left[\nabla^\mu,\Delta\mathcal{L}^{(1)}\right]\right] \\
&+4i\epsilon^{\mu\nu\rho\sigma}v_\rho S_\sigma\left[\overleftarrow{\nabla}_\mu\left[\nabla_\nu\,,\Delta\mathcal{L}^{(1)}\right] - \left[\nabla_\nu\,,\Delta\mathcal{L}^{(1)}\right]\nabla_\mu\right] \\
&+16B\left[\tilde{c}_{13}\left\langle\chi_+\right\rangle^2 + \tilde{c}_{14}\chi_+\left\langle\chi_+\right\rangle + \tilde{c}_{15}\left\langle\chi_+\chi_+\right\rangle\right]\Big\}N'_v\,,
\end{aligned}
\tag{2.40}
$$
where in our case $\Delta\mathcal{L}^{(1)}$ reduces to
$$\Delta\mathcal{L}^{(1)} = e_1\left\langle\chi_+\right\rangle + e_2\left(\chi_+ - \frac{1}{2}\left\langle\chi_+\right\rangle\right)\,, \tag{2.41}$$

and $\Delta\mathcal{L}^{(2)}$ is given by

$$\Delta\mathcal{L}^{(2)} = c_{13}\left\langle\chi_+\right\rangle^2 + c_{14}\chi_+\left\langle\chi_+\right\rangle + c_{15}\left\langle{\chi_+}^2\right\rangle . \quad (2.42)$$

The terms in (2.39) and (2.40) receive contributions from the transformation (2.31) when applied to $\widehat{\mathcal{L}}^{(1)}_{\pi N}$ and to the sum of the (2.28) and (2.29). The $1/m$ expansion contributes as well.

In order to eliminate $\widehat{\mathcal{L}}^{(3)}_{\rm EOM}$ we employ (2.31) with

$$X = \frac{1}{16m^2}\bar{X}\,, \quad Y = \frac{1}{16m^2}\bar{Y} \quad \text{and} \quad Z = \frac{1}{16m^2}\bar{Z}\,.$$

The induced lagrangian is readily obtained:

$$\mathcal{L}_{\rm ind} = \frac{1}{16m^2}\bar{N}'_v\left[-\left(\Delta\mathcal{L}^{(1)}\right)^3 - \frac{1}{2}\left[\Delta\mathcal{L}^{(1)},\left[iv\cdot\nabla,\Delta\mathcal{L}^{(1)}\right]\right] -4\left\{\Delta\mathcal{L}^{(1)},\Delta\mathcal{L}^{(2)}\right\}\right]N'_v\,. \quad (2.43)$$

Adding everything together we find the lagrangian in the scalar sector to $\mathcal{O}(p^3)$

$$\begin{aligned}\widehat{\mathcal{L}}^{(3)}_{\pi N} = \bar{N}_v\Bigg\{&\frac{g_1}{m^2}\left[\nabla_\mu,[\nabla^\mu,\chi_+]\right] + \frac{g_2}{m^2}\left\langle\partial\cdot\partial\,\chi_+\right\rangle \\ &+\frac{g_3}{m^2}\chi_+\left\langle\chi_+\chi_+\right\rangle + \frac{g_4}{m^2}\chi_+\left\langle\chi_+\right\rangle^2 \\ &+\frac{g_5}{m^2}\left\langle\chi_+\right\rangle^3 + \frac{g_6}{m^2}\left\langle\chi_+\chi_+\right\rangle\left\langle\chi_+\right\rangle \\ &+\frac{g_7}{m^2}\left[\chi_+,[iv\cdot\nabla,\chi_+]\right] + \frac{1}{4m^2}\,i\epsilon^{\mu\nu\rho\sigma}v_\rho S_\sigma \\ &\times\Bigg[e_1\left(\overleftarrow{\nabla}_\mu\left\langle\partial_\nu\chi_+\right\rangle - \left\langle\partial_\nu\chi_+\right\rangle\nabla_\mu\right) \\ &+e_2\Bigg(\overleftarrow{\nabla}_\mu\left[\nabla_\nu\,,\chi_+ - \frac{1}{2}\left\langle\chi_+\right\rangle\right] \\ &-\left[\nabla_\nu\,,\chi_+ - \frac{1}{2}\left\langle\chi_+\right\rangle\right]\nabla_\mu\Bigg] + \frac{B}{m^2}\Big[\tilde{c}_{13}\left\langle\chi_+\right\rangle^2 \\ &+\tilde{c}_{14}\chi_+\left\langle\chi_+\right\rangle + \tilde{c}_{15}\left\langle\chi_+\chi_+\right\rangle\Big]\Bigg\}N_v\,. \end{aligned} \quad (2.44)$$

In (2.44) we have subsumed all operators of the form given in (2.35) into the coupling constants g_i. They differ from $\tilde{g}_i$ by a finite renormalization.

3 Mass– and wavefunction renormalization

The formalism presented in Sect. 2 enables us to calculate the T-matrix element of any process with one incoming and one outgoing nucleon in Generalized HBChPT to order p^3. However, as shown by Ecker and Mojžiš, the sources of the heavy component of the nucleon field in the generating functional cannot be dropped altogether. [17] Rather, these terms contribute to the wavefunction renormalization in a non-trivial manner. In order to provide this link between the T-matrix elements calculated in GHBChPT and the relativistic S-matrix elements one is actually seeking, we discuss mass– and wavefunction renormalization to order p^3. The formalism of [17] can be carried over directly to our case and need not to be repeated here. However, we indicate those steps where Generalized ChPT gives raise to new features not present in the standard formulation. In the following we work in the isospin limit.

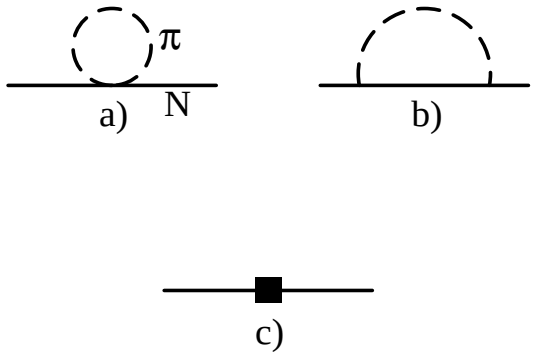

Fig. 1. Feynman diagrams contributing to the self-energy of the nucleon to $\mathcal{O}(p^3)$. Plain and dashed lines denote the nucleon and the pion, respectively. The shaded box denotes the counterterm insertions of order p^2 and p^3

3.1 The nucleon propagator in generalized ChPT

The central object to be considered is the nucleon propagator (*c.f.* [17] for definitions)

$$\begin{aligned}S_N(p) = P_v^+ S_{++} P_v^+ + P_v^+ S_{+-} P_v^- \\ +P_v^- S_{-+} P_v^+ + P_v^- S_{--} P_v^-\end{aligned} \quad (3.1)$$

where the off–shell momentum p is decomposed according to

$$p = mv + k \quad (3.2)$$

with k a residual small momentum. For the present application, we need the pole-part of the objects S_{ij}, $i,j \in \{+,-\}$.

S_{++} in (3.1) is determined by the selfenergy calculated in GHBChPT

$$S_{++}(k)^{-1} = -\Sigma(k)\,. \quad (3.3)$$

The diagrams which contribute are shown in Fig. 1. Diagram a) is a typical new feature of the generalized framework — in standard HBChPT these diagrams enter first at order p^4. Diagram b) is formally the same as in the standard case, but there are nevertheless some differences. First, the propagator in GHBChPT is modified to

$$S^{\rm GHBChPT} = \frac{i}{v\cdot k - \sigma_0}, \qquad \sigma_0 \equiv -4e_1\hat{m}\,. \quad (3.4)$$

We will later see that the shift σ_0 in the propagator gives raise to higher order contributions only and has no net

effect at order p^3. Second, the pion mass entering via the pion propagator is given by the leading order expression in (2.7). Here and below, the symbol M_π^2 is always understood to be defined by that equation. Finally, the contact terms of diagram c) can be read off from (2.33) and (2.44). Explicitly, the selfenergy is found to be

$$\Sigma(k) = \Sigma_{\rm loop}(v\cdot k - \sigma_0) + \Sigma_{\rm contact} \tag{3.5}$$

with

$$\Sigma_{\rm loop}(\omega) = \Sigma^{(a)} + \Sigma^{(b)}(\omega) \tag{3.6}$$

and

$$\Sigma^{(a)} = -\frac{3}{2}\frac{\sigma_0}{F^2}\frac{1}{i}\Delta(M_\pi^2) \tag{3.7}$$

$$\Sigma^{(b)}(\omega) = \frac{3}{4}\frac{g_A^2}{F^2}\left(-\omega\frac{1}{i}\Delta(M_\pi^2) + \left[M_\pi^2 - \omega^2\right]J_0(\omega)\right) . \tag{3.8}$$

The functions $\frac{1}{i}\Delta$ and J_0 are standard one-loop integrals given in Appendix B. The contact contributions of Fig. 1c have the simple form

$$\Sigma_{\rm contact} = -\frac{k^2}{2m} + \Sigma_{\rm CT}^{(2)} + \Sigma_{\rm CT}^{(3)} \tag{3.9}$$

where

$$\begin{aligned}\Sigma_{\rm CT}^{(2)} &= -\frac{8\hat{m}^2}{m}\left(2f_{13} + f_{14} + f_{15}\right)\\ \Sigma_{\rm CT}^{(3)} &= -\frac{16\hat{m}^3}{m^2}\left(g_3 + 2g_4 + 4g_5 + 2g_6\right)\\ &\quad -\frac{8\hat{m}^2 B}{m^2}\left(2\tilde{c}_{13} + \tilde{c}_{14} + \tilde{c}_{15}\right) .\end{aligned} \tag{3.10}$$

The loop contributions to the selfenergy contain divergences which, for physical quantities, can always be absorbed by a appropriate renormalization of the the counterterms g_i and $\tilde{c}_i$ in $\Sigma_{\rm CT}^{(3)}$. This is discussed in Sect. 3.2 below.

The pole-part of S_{+-}, S_{-+} and S_{--} is proportional to S_{++}. To the order we are working we have

$$\begin{aligned}S_{+-}(k) &= \frac{1}{2m}P_v^+ S_{++}\left(1 - \frac{v\cdot k + \sigma_0}{2m} + \mathcal{O}(p^3)\right)\not{k}^\perp P_v^-\\ S_{-+}(k) &= \frac{1}{2m}P_v^- S_{++}\left(1 - \frac{v\cdot k + \sigma_0}{2m} + \mathcal{O}(p^3)\right)\not{k}^\perp P_v^+\\ S_{--}(k) &= \frac{1}{(2m)^2}P_v^-\ \not{k}^\perp S_{++}\ \not{k}^\perp P_v^- + \mathcal{O}(p^3) .\end{aligned} \tag{3.11}$$

Due to the presence of the projection operators P_v^- the term in proportion to S_{--} is suppressed by one additional power of p and does not contribute at the order we are considering here.

Following again [17] we now define the on-shell nucleon momentum p_N as

$$\begin{aligned}p &= mv + k \equiv p_N + \lambda r\\ p_N &\equiv m_N v + Q .\end{aligned} \tag{3.12}$$

The arbitrary four-vector r controls the on-shell limit $p \to p_N$ by letting the real parameter λ tend to zero. We can choose $r = v$ for convenience. The nucleon mass has the expansion

$$m_N = m + \sigma_0 + \delta m^{(2)} . \tag{3.13}$$

Here we have displayed explicitly the leading order correction σ_0. As emphasized previously we count σ_0 as a quantity of order p. The remainder, $\delta m^{(2)}$, is of $\mathcal{O}(p^2)$ by definition. Then we have

$$p_N^2 = m_N^2 \quad\Longrightarrow\quad 2m_N v\cdot Q + Q^2 = 0 \tag{3.14}$$

and

$$v\cdot k = \sigma_0 + \delta m^{(2)} - \frac{Q^2}{2m} + \lambda . \tag{3.15}$$

We observe that on-shell ($\lambda \to 0$)

$$v\cdot k - \sigma_0 = \mathcal{O}(p^2) , \tag{3.16}$$

which will be crucial for the analysis to follow[3].

3.2 The nucleon mass to $\mathcal{O}(p^3)$

The pole of the nucleon propagator is entirely determined by S_{++}. On-shell, and to $\mathcal{O}(p^3)$, we have

$$\begin{aligned}S_{++}^{(-1)}(k) &= v\cdot k - \sigma_0 - \Sigma(k)\\ &= \frac{1}{2m}\Big(p^2 - m^2 - 2m\Big[\sigma_0 + \Sigma_{\rm CT}^{(2)}\\ &\quad + \Sigma_{\rm CT}^{(3)} + \Sigma_{\rm loop}(v\cdot k - \sigma_0)\Big]\Big)\end{aligned} \tag{3.17}$$

Expanding $\Sigma_{\rm loop}$ according to

$$\begin{aligned}&\Sigma_{\rm loop}(v\cdot k - \sigma_0)\\ &\quad = \Sigma_{\rm loop}(0) + (v\cdot k - \sigma_0)\Sigma'_{\rm loop}(0) + \ldots\end{aligned} \tag{3.18}$$

we observe that due to (3.16) only the leading term in this expansion has to be kept. Thus

$$m_N^2 = m^2 + 2m\left[\sigma_0 + \Sigma_{\rm CT}^{(2)} + \Sigma_{\rm CT}^{(3)} + \Sigma_{\rm loop}(0)\right] . \tag{3.19}$$

The next step consists of removing the divergences in $\Sigma_{\rm loop}(0)$ by renormalization of the counter term coupling constants. Employing (3.6) the divergent part of the one-loop selfenergy can be written as

$$\Sigma_{\rm loop}(0)_{|\ \rm div} = \frac{24e_1}{F^2}\left(B\hat{m}^2 + 2A\hat{m}^3\right)\cdot\Lambda(\mu) \tag{3.20}$$

[3] The propagator in GHBChPT depends also only on the combination $v\cdot k - \sigma_0$. When doing loop calculations, this can be used to show that, to leading order, the shift σ_0 in the propagator has no net effect. This result could be obtained alternatively by choosing a shifted mass $m \to m + \sigma_0$ in the exponential factor occurring in the definition of the heavy baryon field, *c.f.* (2.16). However, we prefer to keep the mass correction σ_0 explicit, since it depends on the light quark mass

where $\Lambda(\mu)$ contains the pole for $d \to 4$ and is defined in Appendix B. Defining renormalized couplings

$$\begin{aligned} g_i &= g_i^r(\mu) + \frac{m^2}{F^2}\beta_{g_i} \cdot \Lambda(\mu) \\ \tilde{c}_i &= \tilde{c}_i^r(\mu) + \frac{m^2}{F^2}\beta_{\tilde{c}_i} \cdot \Lambda(\mu) \end{aligned} \tag{3.21}$$

the nucleon mass in (3.19) is rendered finite for

$$\begin{aligned} 2\beta_{\tilde{c}_{13}} + \beta_{\tilde{c}_{14}} + \beta_{\tilde{c}_{15}} - 3e_1 &= 0 \\ \beta_{g_3} + 2\beta_{g_4} + 4\beta_{g_5} + 2\beta_{g_6} - 3e_1 A &= 0\,. \end{aligned} \tag{3.22}$$

The constants f_i need not to be renormalized.

Finally we solve (3.19) for m_N. Neglecting consistently higher order terms we find

$$m_N = m + \Delta m^{(1)} + \Delta m^{(2)} + \Delta m^{(3)} + \mathcal{O}(p^4) \tag{3.23}$$

with

$$\begin{aligned} \Delta m^{(1)} &= \sigma_0 \\ \Delta m^{(2)} &= -\frac{8\hat{m}^2\,(2f_{13} + f_{14} + f_{15}) + \frac{1}{2}\sigma_0^2}{m_N} \\ \Delta m^{(3)} &= -\frac{16\hat{m}^3}{m_N^2}\,(g_3^r + 2g_4^r + 4g_5^r + 2g_6^r) \\ &\quad -\frac{8\hat{m}^2 B}{m_N^2}\,(2\tilde{c}_{13}^r + \tilde{c}_{14}^r + \tilde{c}_{15}^r) \\ &\quad -\frac{3\sigma_0 M_\pi^2}{32\pi^2 F_\pi^2}\ln\frac{M_\pi^2}{\mu^2} - \frac{3\mathring{g}_A^2}{32\pi F_\pi^2}M_\pi^3\,. \end{aligned} \tag{3.24}$$

We have expressed everything in terms of the physical nucleon mass m_N. In standard HBChPT only σ_0 and the term in proportion to M_π^3 enter at $\mathcal{O}(p^3)$. The additional terms in proportion to f_i, g_i and $\tilde{c}_i^r$ are not known and prevent us from giving a quantitative estimate of the nucleon mass shift[4]. However, the formula can be used in further applications of GHBChPT and we hope that the unknown constants can be determined in such future work.

3.3 Wavefunction renormalization to $\mathcal{O}(p^3)$

The wavefunction renormalization constant Z_N is defined via

$$\begin{aligned} Z_N(Q)u(p_N) &= \lim_{p\to p_N} S_N(p)(\not{p} - m_N)u(p_N) \\ &= \lim_{\lambda\to 0} S_N(p)\lambda\ \not{v}u(p_N)\,. \end{aligned} \tag{3.25}$$

Employing (3.15,3.16) and keeping terms linear in λ only we may expand

$$\Sigma_{\rm loop}(v\cdot k - \sigma_0) = \Sigma_{\rm loop}(0) + \lambda\Sigma'_{\rm loop}(0) + \mathcal{O}(\lambda^2, p^4)\,. \tag{3.26}$$

[4] Only the combinations $f_i + \frac{B}{m_N}\tilde{c}_i^r$, $i = 13, 14, 15$ enter the expression and can possibly be fixed from experiment

Thus

$$Z_N(Q)u(p_N) = \frac{m\left(P_v^+ + \frac{1}{2m}\ \not{k}^\perp\left[1 - \frac{v\cdot k + \sigma_0}{2m}\right]\right)\ \not{v}u(p_N)}{v\cdot p_N - m\Sigma'_{\rm loop}(0)}\,. \tag{3.27}$$

The same steps as performed in [17] but keeping track of the peculiarities due to GHBChPT lead to the final result

$$Z_N(Q) = 1 - \frac{1}{m_N}\left(\sigma_0 + \Delta m^{(2)}\right) + \frac{Q^2}{4m_N^2} + \Sigma'_{\rm loop}(0) \tag{3.28}$$

where

$$\Sigma'_{\rm loop}(0) = -\frac{9}{2}\frac{\mathring{g}_A^2 M_\pi^2}{16\pi^2 F_\pi^2}\left(16\pi^2\Lambda(\mu) + \ln\frac{M_\pi}{\mu} + \frac{1}{3}\right)\,. \tag{3.29}$$

This result is identical to the result obtained in standard HBChPT except for the appearance of the term in proportion to $\Delta m^{(2)}$. The fact that the wavefunction renormalization "constant" depends on the momentum Q is not affected. As before we have expressed all contributions in terms of the physical nucleon mass.

4 The scalar form factor of the nucleon

As a further application of our formalism we consider the scalar form factor of the nucleon

$$\left\langle \Psi(p')|\hat{m}\ \left(\bar{u}u + \bar{d}d\right)|\Psi(p)\right\rangle = \sigma(t)\bar{u}(p')u(p)\,, \tag{4.1}$$

which is a measure of explicit chiral symmetry breaking due to up and down quark mass. The variable $t = (p'-p)^2$ denotes the square of the four-momentum transfer. At $t = 0$ the scalar form factor yields the so called sigma term of the nucleon, which has attracted much attention over the years. The interest in this quantity derived partly from the discrepancy between early determinations of $\sigma(0)$ from πN-scattering data and naive estimates based on the analysis of the baryon mass spectrum. Gasser, Leutwyler and Sainio have resolved the issue by a thorough dispersive analysis of the nucleon sigma term. [24] The method relies on three steps: i) a low energy theorem due to Brown, Pardee and Peccei [25] relates the isospin even πN-scattering amplitude (with Born term removed) at the Cheng-Dashen point, $\bar{D}^+(2M_\pi^2)$, to the scalar form factor of the nucleon, *i.e.*

$$\Sigma \equiv F_\pi^2\bar{D}^+(2M_\pi^2) = \sigma(2M_\pi^2) + \Delta_{\rm R}\,. \tag{4.2}$$

The remainder $\Delta_{\rm R}$ is of $\mathcal{O}(\hat{m}^2)$ and was recently calculated to order p^4 in standard HBChPT. [26] It was shown that potentially large contributions due to chiral logarithms of the form $M_\pi^4\ln M_\pi$ cancel exactly; $\Delta_{\rm R}$ is indeed small and was estimated to be bounded by 2 MeV. ii) Σ is determined by the extrapolation of the πN scattering amplitude from the physical region $t \le 0$ to the Cheng-Dashen point. It is useful to decompose

$$\Sigma = \Sigma_{\rm d} + \Delta_{\rm D}\,, \tag{4.3}$$

where $\Sigma_{\rm d}$ denotes the first two terms in a Taylor series expansion. The point is that $\Sigma_{\rm d}$ is fixed in terms of the πN scattering amplitude in the physical region. The remainder, $\Delta_{\rm D}$, is also accessible through a dispersive analysis but depends on the $\pi\pi$ phase shift. It is particularly sensitive to the region just above the two pion threshold. Numerically, the analysis in [27] yielded $\Sigma_{\rm d} = 48$ MeV, with an error bar of about 8 MeV, and $\Delta_{\rm D} = 11.9 \pm 0.6$ MeV. We refer to [24,27,28] for a detailed account on this part. iii) In order to determine the nucleon sigma term from (4.2), it remains to calculate the shift of the scalar form factor from the Cheng-Dashen point to zero momentum transfer,

$$\Delta_\sigma = \sigma(2M_\pi^2) - \sigma(0)\,. \tag{4.4}$$

In [27] Δ_σ was calculated by means of a once subtracted dispersion relation for $\sigma(t)$, yielding

$$\Delta_\sigma = 15.2 \pm 0.4\ \text{MeV}\,. \tag{4.5}$$

It was noted that Δ_σ cannot be reliably calculated in chiral perturbation theory, say to one-loop. The reason is that the imaginary part entering the dispersion relation is in proportion to $\Gamma_\pi(t) = \langle \pi^0(p')|\hat{m}(\bar{u}u + \bar{d}d|\pi^0(p)\rangle$, the scalar form factor of the pion, and to f_+^0, the I=J=0 πN partial wave in the t-channel. However, both are grossly underestimated in the two-pion threshold region if leading order ChPT calculations are employed.

If the quark condensate is substantially smaller than assumed in standard ChPT, all of the three steps mentioned above are subject to modifications and must be reanalyzed. In step ii) and iii), the $\pi\pi$ phase shift close to threshold plays an important role. Moreover, the normalization of the pion scalar form factor, which enters the determination of Δ_σ, is also sensitive to the light quark condensate[5]. This can be seen best by using the Feynman-Hellman theorem for $\Gamma_\pi(0)$, *i.e.*

$$\Gamma_\pi(0) = \hat{m}\frac{\partial M_\pi^2}{\partial \hat{m}} = 2B\hat{m} + 8A\hat{m}^2 \equiv M_\pi^2(2-x)\,, \tag{4.6}$$

with

$$x \equiv x_{\rm GOR} = \frac{2B\hat{m}}{M_\pi^2}\,, \qquad 0 \le x \le 1\,. \tag{4.7}$$

The parameter x interpolates between the extreme generalized limit, $x = 0$, and the standard case with $x = 1$. Note that x is given in terms of B, a quantity of chiral $SU(2)$. The relation between x and the more familiar ratio $r = m_s/\hat{m}$ is given in [29]. For $r \lesssim 12$ the normalization of the pion scalar form factor starts to deviate strongly from the standard case. We will come back to a dispersive treatment of $\sigma(t)$ in Sect. 4.2 but now turn to the GHBChPT calculation.

4.1 Scalar form factor of the nucleon to $\mathcal{O}(p^3)$ in GHBChPT

Although it is clear from the above that ChPT itself cannot provide a full understanding of the nucleon sigma term, ChPT is nevertheless needed in order to provide important constraints on the dispersive analysis. The method of calculating fully relativistic quantities like the scalar form factor in (4.1) by using HBChPT has been spelled out in [17]. We follow this method and also employ the initial nucleon rest frame (INRF) when applying wave function renormalization. The fully renormalized scalar form factor thus calculated in HBChPT coincides with the relativistic form factor we are seeking. The Feynman diagrams contributing to order p^3 GHBChPT are shown in Fig. 2. Compared to the standard case there are three additional loop diagrams, *i.e.* graphs b), c) and e). In dimensional regularization the sum of all loop graphs is found to be

$$\begin{aligned}\sigma(t)_{\rm loop} = &-\frac{3}{2}\sigma_0\frac{\Delta(M_\pi^2)}{F^2} + \frac{3\mathring{g}_A^2\sigma_0}{4F^2}\left[\Delta(M_\pi^2) - M_\pi^2 J_0'(0)\right] \\ &+\frac{3}{2}(2-x)\,\sigma_0\frac{M_\pi^2}{F^2}J_{\pi\pi}(t) - \frac{3}{8}(2-x)\,\frac{\mathring{g}_A^2 M_\pi^2}{F^2} \\ &\times\left[\left(t-2M_\pi^2\right)K_0(0,t) - 2J_0(0)\right]\,,\end{aligned} \tag{4.8}$$

where σ_0 is the lowest order contribution to $\sigma(t)$

$$\sigma_0 = -4\hat{m}e_1\,. \tag{4.9}$$

The loop functions Δ, J_0', $J_{\pi\pi}$, K_0, and J_0 are given in Appendix B.

The tree-contributions in Fig. 2a are modified as well. Besides the $\mathcal{O}(p)$ contribution, which in the standard case

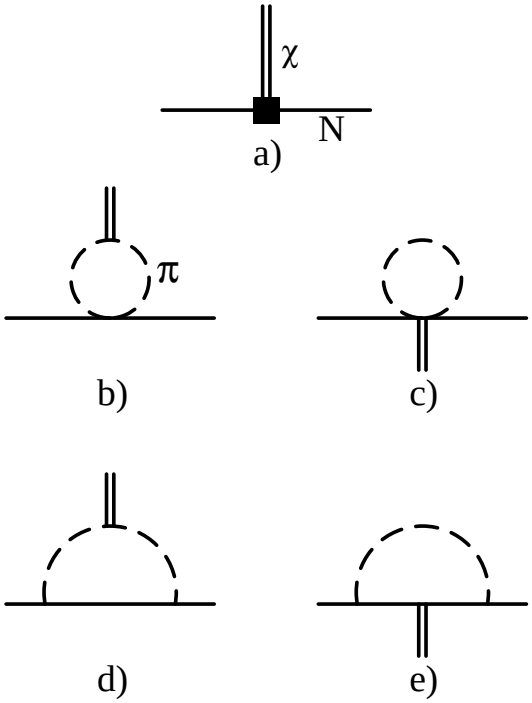

Fig. 2. Feynman diagrams contributing to the scalar form factor of the nucleon to $\mathcal{O}(p^3)$. Plain, dashed, and double lines line denote the nucleon, the pion, and the scalar source, respectively. The shaded box denotes tree contributions of order p, p^2, and p^3

[5] We are grateful to Jan Stern for pointing this out to us

was counted as $\mathcal{O}(p^2)$, there are genuine new vertices of the order p^2 and p^3. These can be read off easily from the corresponding effective lagrangians given in Sect. 2. We find

$$\begin{aligned}\sigma_{\rm tree}(t) &= \sigma_0 - \frac{16\hat{m}^2}{m}\left[2f_{13}+f_{14}+f_{15}+\frac{B}{m}\left(2\tilde{c}_{13}+\tilde{c}_{14}+\tilde{c}_{15}\right)\right] \\ &\quad -\frac{48\hat{m}^3}{m^2}\left(g_3+2g_4+4g_5+2g_6\right)+\frac{\hat{m}t}{m^2}\left(g_1+2g_2\right)\,. \end{aligned} \tag{4.10}$$

Finally we must take into account wave-function renormalization of the in- and outgoing fields N_v and $\bar{N}_v$, respectively,

$$\sigma(t) \;= (\sigma_{\rm tree}(t)+\sigma_{\rm loop}(t))\sqrt{Z_N(0)Z_N(Q)}\,. \tag{4.11}$$

As mentioned above, we work in the INRF where incoming and outgoing nucleon four momentum are given as

$$p_{\rm in} = m_N v, \qquad p_{\rm out} = m_N v + Q\,, \quad Q = p - p'\,. \tag{4.12}$$

For the following it is convenient to decompose the scalar form factor according to

$$\sigma(t) \equiv \sigma(0) + \bar{\sigma}(t) \tag{4.13}$$

and to discuss the two pieces separately. The contributions to $\sigma(0)$ of the loop-graphs as well as those from wavefunction renormalization contain divergences for $d \to 4$. These are removed by introducing renormalized coupling constants g_i^r and $\tilde{c}_i^R$ as given in (3.21,3.22). Using the explicit form of the loop functions at zero momentum transfer and expanding consistently up to $\mathcal{O}(p^3)$, we arrive at the final result for the nucleon sigma term

$$\begin{aligned}\sigma(0) = \sigma_0 &+ \left(2-\frac{\sigma_0}{m_N}\right)\Delta m^{(2)} \\ &-\frac{16\hat{m}^2 B}{m_N^2}\left(2\tilde{c}_{13}^r+\tilde{c}_{14}^r+\tilde{c}_{15}^r\right) \\ &-\frac{48\hat{m}^3}{m_N^2}\left(g_3^r+2g_4^r+4g_5^r+2g_6^r\right) \\ &-\frac{3\sigma_0 M_\pi^2}{32\pi^2F_\pi^2}\left[(3-x)\ln\frac{M_\pi^2}{\mu^2}+(2-x)\right] \\ &-\frac{9\mathring{g}_A^2}{64\pi F_\pi^2}(2-x)M_\pi^3\,, \end{aligned} \tag{4.14}$$

where $\Delta m^{(2)}$ was given in (3.24). This result agrees with the Feynman-Hellman theorem

$$\sigma(0) = \hat{m}\frac{\partial}{\partial\hat{m}}m_N \tag{4.15}$$

and therefore provides a nice check on our calculation.

The t–dependent part of the scalar form factor involves only finite loop functions and needs no infinite renormalization of counterterms. We thus obtain the scale independent result

$$\begin{aligned}\bar{\sigma}(t) = &\frac{\sigma_0+8\hat{m}(g_1+2g_2)}{8m_N^2}\cdot t+\frac{3\sigma_0 M_\pi^2}{2F_\pi^2}(2-x)\bar{J}_{\pi\pi}(t) \\ &-\frac{3\mathring{g}_A^2 M_\pi^2}{8F_\pi^2}(2-x)\left[(t-2M_\pi^2)K_0(0,t)-\frac{M_\pi}{8\pi}\right] \end{aligned} \tag{4.16}$$

The loop contributions scale with $(2-x)$, *i.e.* there is a factor of two difference between the extreme generalized ($x=0$) and standard case ($x=1$). The polynomial part linear in t on the other hand does not exhibit this scale factor. The combination of coupling constants g_1+2g_2 is unknown; in standard ChPT these terms would occur at order p^4.

The phenomenological implications of these results can be assessed only in comparison with the dispersive analysis as described in [24,27] for the standard case and outlined at the beginning of this section. As to $\sigma(0)$, we do not know the coupling constants e_1, $\tilde{c}_i^r$, and g_i^r. On dimensional grounds, we expect these constants to be of order unity. The leading order term, σ_0, can be estimated as follows. The counter term contributions of $\mathcal{O}(p^2)$ are suppressed by additional factors $\hat{m}/m_N$ and therefore must be small. The loop contributions, *i.e.* the last line in (4.14), depend explicitly on the ratio x. The term in proportion to $\mathring{g}_A^2$ is dominating and numerically yields $-(2-x)22.5$ MeV. Once the sigma term is determined from a dispersive analysis, we then have

$$\sigma_0 \approx \sigma(0)^{\rm dispersive} + (2-x)\,22.5\,{\rm MeV}. \tag{4.17}$$

Note that the result of the dispersive analysis will also depend on x. Incidentally, this variation with x partially cancels the x-dependence of the second term in (4.17), leading to $\sigma_0 \approx 67.5\ldots80$ MeV, where the lower and upper bound correspond to $x=1$ and $x=0$, respectively. [30]

The phenomenological analysis of the t-dependent part is also instructive. Consider the shift between Cheng-Dashen point and zero momentum transfer to $\mathcal{O}(p^3)$

$$\begin{aligned}\Delta_\sigma = &\frac{2M_\pi^3}{m_N^2}\left[\frac{\hat{m}}{M_\pi}\left(g_1^r+2g_2^r\right)+\frac{\sigma_0}{8M_\pi}\right] \\ &+(2-x)\,\frac{2M_\pi^3}{(4\pi F)^2}\frac{3\pi}{8}\left[\mathring{g}_A^2+\frac{\sigma_0}{M_\pi}\left(\frac{4}{\pi}-1\right)\right]\,. \end{aligned} \tag{4.18}$$

The second term in the first square bracket is due to wave-function renormalization. The standard result to order $\mathcal{O}(p^3)$ is obtained by setting $x=1$ and by the observation that in this case all terms in proportion to σ_0 and the couplings g_i^r are at least of $\mathcal{O}(p^4)$. Setting $\mathring{g}_A = 1.26$, $F_\pi = 92.4$ MeV, $m_N = 939$ MeV we obtain

$$\begin{aligned}\Delta_\sigma = \Bigg\{&6\left[\frac{\hat{m}}{M_\pi}\left(g_1^r+2g_2^r\right)+\frac{\sigma_0}{8M_\pi}\right] \\ &+(2-x)\left[7.5+1.6\frac{\sigma_0}{M_\pi}\right]\Bigg\}\,{\rm MeV}\,. \end{aligned} \tag{4.19}$$

In order to get a rough estimate of the size of the contribution not in proportion to $(2-x)$ we set $\sigma_0 = 70$ MeV and employ a typical value for the light quark mass in generalized ChPT, $\hat{m} = 20$ MeV. On dimensional analysis grounds one expects the coupling constants g_1^r and g_2^r to be of order unity. Varying the sum $g_1^r+2g_2^r$ between the bounds ±3 yields a net contribution of about ±3 MeV to Δ_σ. The second term in (4.19) is due to finite loop graphs.

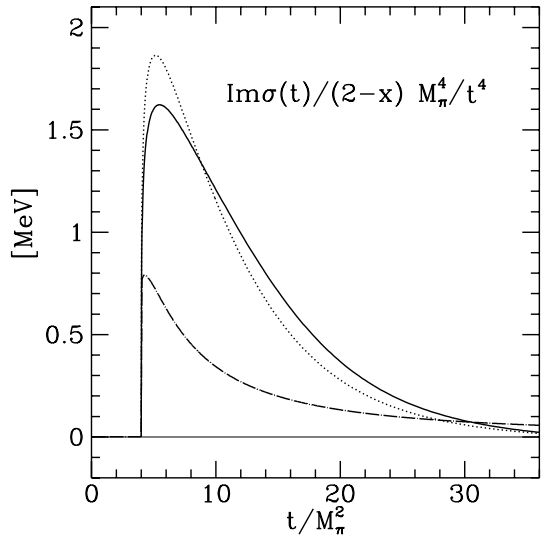

Fig. 3. The imaginary part of the scalar form factor. Full, dotted and dashed-dotted lines denote the dispersive analysis in the standard case, the dispersive analysis in the extreme generalized case and the $\mathcal{O}(p^3)$ GHBChPT calculation, respectively

Numerically, the term in proportion to $\mathring{g}_A^2$ is the dominant contribution for reasonable values of σ_0, i.e. $\sigma_0 = 60 \ldots 80$ MeV. Due to the scale factor $(2-x)$ the shift Δ_σ yields contributions which can be larger than the standard ChPT result to $\mathcal{O}(p^3)$ by more than a factor of 2.

How does this result compare to a dispersive analysis adopted to the case of a small quark condensate? While a full treatment is outside the scope of this article, we sketch in Sect. 4.2 below a dispersive treatment of $\bar{\sigma}(t)$. Details are deferred to a forthcoming publication. [30] In Fig. 3 we compare the imaginary part of $\sigma(t)$ of the dispersive analysis with the result of the $\mathcal{O}(p^3)$ GHBChPT calculation. The dispersive result is seen to yield a strong enhancement over the ChPT calculation, as pointed out in [27]. The figure clearly shows that a leading order ChPT calculation is not appropriate to calculate the imaginary part reliably, both in the standard as well as in the generalized case. Note that the universal factor $(2-x)$ has been divided out. The comparison also shows that the failure of the GHBChPT calculation for Δ_σ (compare Table 1) cannot be blamed on the dimensional estimate for the coupling constant $g_1^r + 2g_2^r$. We know that higher order corrections in the chiral expansion will modify the imaginary part substantially. Neglecting these and adjusting $g_1^r + 2g_2^r$ such that the dispersive value for Δ_σ is reproduced is therefore without justification.

4.2 Dispersive analysis of $\sigma(t)$

We present a dispersive analysis of $\bar{\sigma}(t)$ adopted to the case of a small quark condensate. The model serves as a representation to which the ChPT calculation can be compared, but also provides a first step towards the calculation of the nucleon sigma term. We employ the once subtracted dispersion relation

$$\bar{\sigma}(t) = \frac{t}{\pi} \int dt' \frac{\mathrm{Im}\,\sigma(t')}{t'(t'-t-i\epsilon)} \tag{4.20}$$

and use the imaginary part as given in the elastic region $4M_\pi^2 < t < 16M_\pi^2$ via

$$\mathrm{Im}\,\sigma(t) = \frac{3}{2} \frac{\Gamma_\pi^*(t) f_+^0(t)}{4m_N^2 - t} \left(1 - \frac{4M_\pi^2}{t}\right)^{\frac{1}{2}} . \tag{4.21}$$

Here, $\Gamma_\pi(t)$ is the scalar form factor of the pion and $f_+^0(t)$ is the I=J=0 πN partial wave in the t-channel. Both of these amplitudes are subject to strong final state interactions of the two pions. In order to account for these effects, we model the imaginary part as follows. For the pion form factor, we use the form

$$\Gamma_\pi(t) = \Gamma_\pi(0)(1 + b \cdot t) \exp\{\Delta_0(t)\} , \tag{4.22}$$

where

$$\Delta_0(t) = \frac{t}{\pi} \int_{4M_\pi^2}^{t_1} dt' \frac{\delta_0^0(t')}{t'(t'-t-i\epsilon)} \tag{4.23}$$

is referred to as the Omnès function and δ_0^0 denotes the I=J=0 $\pi\pi$ phase shift. For definiteness, the cutoff in (4.23) is taken as $t_1 = (0.9\,\mathrm{GeV})^2$.

The generalized scenario of SBχS enters (4.22) in two ways. First, the normalization $\Gamma_\pi(0)$ is sensitive to the quark condensate, and to leading order it is given by (4.6). Higher order corrections are expected to be small and we employ the simple form (4.6) in our dispersive analysis. Second, the I=J=0 $\pi\pi$ phase shift differs considerably from the standard case, in the threshold region. Consequently, the Omnès function is modified, leading to a further enhancement of the pion form factor close to threshold. The polynomial $(1 + b \cdot t)$ in (4.22) is used to mimic the effect of a two-channel analysis which, in the standard case, was seen to modify the pion form factor substantially above $t = (0.45\,\mathrm{GeV})^2$. [31]. We expect these effects to a large extent to be independent of the $\pi\pi$ phase shift at threshold and fix the parameter universally at $b = 0.038\,\mathrm{fm}^2$.

As to f_+^0 we employ the strategy developed in [27, 32] in order to extrapolate to the unphysical region. For

Table 1. r-dependence of Δ_σ and $\frac{d\sigma(0)}{dt}$ normalized to the standard case

α	1	2	2.5	3	3.5	4
$r = \frac{m_s}{\hat{m}}$	≈ 25	12	10.25	9.2	8.5	≈ 8
$2-x$	1	1.40	1.58	1.75	1.93	2.00
$\Delta_\sigma^{\mathrm{gen}}/\Delta_\sigma^{\mathrm{stan}}$	1	1.38	1.55	1.71	1.87	1.93
$\frac{d\sigma^{\mathrm{gen}}(0)}{dt}/\frac{d\sigma^{\mathrm{stan}}(0)}{dt}$	1	1.37	1.53	1.68	1.84	1.89

$t \leq 0$ we use the data tabulated in [33]. The continuation depends on the $\pi\pi$ phase shift employed – here enters the assumption made about the quark condensate. We use a parameterization of δ_0^0 given by Schenk [34] with threshold parameters a_0^0, b_0^0 taken from [29]. Details of this analysis will be presented elsewhere [30].

The model specified by (4.20-4.23) and the continuation of f_+^0 just described is then used to calculate $\bar{\sigma}(t)$ for $0 < x < 1$. In Fig. 3 the imaginary parts of $\sigma(t)$ for the two limiting cases are compared. The threshold enhancement of the curve corresponding to the dispersive treatment of the extreme generalized case ($x = 0$) over that of the standard case ($x = 1$) is due to the larger scattering length of the former. The area under the two dispersive curves in Fig. 3 is roughly equal, however. The ratio $\Delta_\sigma^{\text{generalized}}(x)/\Delta_\sigma^{\text{standard}}$ is thus close to $(2 - x)$. In Table 1 we give this ratio as well $d\sigma(0)/dt$ normalized to the standard case for various values of x. α is related to the $\pi\pi$ scattering amplitude at the symmetric point $s = t = u$. [8] For the sake of comparison we also give the corresponding values of r, taken from [29], and $2 - x$. The shift of scalar form factor of the nucleon deviates from the standard case substantially for $r \lesssim 12$.

We close the discussion with a remark concerning the dispersive analysis of the nucleon sigma term itself. As mentioned at the beginning of this section, $\sigma(0)$ can be obtained by updating the estimates for Δ_{R} and Δ_{D}, adopted to the case of a small quark condensate. Δ_{D} is particularly sensitive to the threshold behaviour of the $\pi\pi$ phase shift. The crucial question here is whether the almost perfect cancellation between the remainders Δ_{D} and Δ_σ observed in [24] persist in the case of a small quark condensate. Work in this direction is under way and results will be reported in [30].

5 Conclusions

The consequences of a small quark condensate are studied for the baryonic sector of ChPT. To this end, we have constructed an effective theory of the πN–system respecting chiral symmetry and admitting a systematic expansion in small momenta, the light quark masses, and the dimensionful parameter $B = -\langle\bar{q}q\rangle/F^2$, collectively denoted as p (GHBChPT). The light quark masses are counted as order p, in contrast to the standard counting rule $m_q \sim p^2$. Moreover, we assume that in the chiral limit the theory contains no other small scales than B. The effective lagrangian is given in it's most general form to $\mathcal{O}(p^2)$ and to $\mathcal{O}(p^3)$ in the scalar sector. A method to efficiently construct the relativistic baryonic chiral lagrangians for chiral SU(2) to all orders is given in the Appendix.

Mass- and wavefunction renormalization have been calculated to $\mathcal{O}(p^3)$. These results will be useful for future applications of the formalism laid out in this article. We have, then, considered the scalar form factor of the nucleon to order p^3. The result depends on additional low energy coupling constants not present in the standard case at this order. By comparison to a dispersive treatment of the subtracted nucleon scalar form factor adopted to the generalized scenario of SBχS, it is shown that the chiral prediction for the shift $\Delta_\sigma = \sigma(2M_\pi^2) - \sigma(0)$ is unreliable also in generalized ChPT. Moreover, the dispersive analysis yields a strong deviation of Δ_σ from the standard result provided $r = m_s/\hat{m} \lesssim 12$, which can reach up to a factor of two for the limiting case of a vanishing quark condensate. In order to determine the nucleon sigma term, both the remainder at the Cheng-Dashen point, Δ_{R}, as well as the remainder in the extrapolation of the πN scattering amplitude from the physical region to the Cheng-Dashen point, Δ_{D}, have to be reanalyzed without the assumption of a large quark condensate.

Other processes like πN-scattering or $\pi N \to N\pi\pi$ are expected to be sensitive to the value of the light quark condensate too. We hope that future studies in the framework presented here will lead to a determination of many of the unknown coupling constants. This, together with dispersive theoretic methods, should ultimately make it possible to test the standard scenario of spontaneous breakdown of chiral symmetry in the baryonic sector as well.

Acknowledgements. We would like to thank M. Knecht for correspondence and J. Stern for discussion initiating our work on the dispersive treatment of the nucleon scalar form factor. This work was supported in part by Schweizerischer Nationalfonds.

A Construction of a $SU(2)$ invariant Lagrangian

In the following we will give a recipe of how to efficiently construct the relativistic baryon-meson Lagrangian $\mathcal{L}_{\pi N}$. We demand that $\mathcal{L}_{\pi N}$ be hermitian, flavor neutral, invariant under $SU(2)$ chiral transformations, proper Lorentz transformations, and the discrete symmetries[6] C,P, and T. The method follows closely that used by Krause for the case of chiral $SU(3)$ [35]. There are some differences, however, which are due to the fact that in chiral $SU(2)$ the Baryons belong to the fundamental representation. Also, we feel an explicit exposition of the rules employed will be helpful for future work in HBCHPT, both in the standard as well as in the generalized version. An alternative derivation has appeared recently in [36].

The construction of the effective Lagrangian is built upon the traceless chiral fields

$$u_\mu,\ f_\pm^{\mu\nu},\ \chi_\pm - \frac{1}{2}\langle\chi_\pm\rangle\ , \tag{A.1}$$

and the singlet chiral fields

$$\langle\chi_\pm\rangle\ , v_{\mu\nu}^{(s)}\ . \tag{A.2}$$

All of these are 2×2 matrices in flavor space. In order to keep the following exposition and expressions as simple as possible, we slightly change our notation of the covariant

[6] The transformation properties of the chiral fields and of the elements of the Clifford Algebra are listed in Table 2 and Table 3 respectively

derivative. In the previous sections a covariant derivative acted on all fields to the right of it. Now it is understood that a derivate acts only on the field right next to it, *i.e.*

$$\nabla_\mu X = \partial_\mu X + [\Gamma_\mu, X] \; . \tag{A.3}$$

The covariant derivative on the nucleon field Ψ is given by

$$\nabla_\mu \Psi = \partial_\mu \Psi + \left(\Gamma_\mu - i v_\mu^{(s)} \right) \Psi \, . \tag{A.4}$$

The chiral order of the fields are:

$$\chi_\pm \, , \; \chi_\pm - \frac{1}{2} \langle \chi_\pm \rangle \, , \; u_\mu \sim \mathcal{O}(p) \, , \quad f_\pm^{\mu\nu} \, , \; v_{\mu\nu}^{(s)} \sim \mathcal{O}(p^2) \, . \tag{A.5}$$

A covariant derivative acting on these fields increases the chiral order by one. The covariant derivative on the nucleon field Ψ counts as

$$\nabla_\mu \Psi \sim \mathcal{O}(1) \, . \tag{A.6}$$

In what follows we will construct the most general operator which can occur in $\mathcal{L}_{\pi N}$. In a first step we restrict ourselves to terms in $\mathcal{L}_{\pi N}$ without derivatives acting on the nucleon fields Ψ or $\bar\Psi$. In this case such a term is generically of the form

$$\bar\Psi \Gamma A \Psi \, , \qquad \Gamma \in \text{Clifford Algebra} \, . \tag{A.7}$$

The term A is a polynomial in the chiral fields of (A.1), (A.2), and covariant derivatives thereof. To ensure Lorentz invariance all Lorentz indices must be contracted with a suitable combination of the metric $g^{\mu\nu}$ or the completely antisymmetric tensor $\epsilon^{\mu\nu\rho\sigma}$. Instead of writing the polynomial A as a simple product of chiral fields it is more suitable to consider the general polynomial A in the somewhat more complex form

$$A = (A_1, (A_2, \cdots A_n) \dots) \tag{A.8}$$

where (A_1, A_2) denotes either the commutator $[A_1, A_2]$ or the anticommutator $\{A_1, A_2\}$ of the chiral fields A_1 and A_2. The polynomial A can then be simplified with the observation that

$$[D, [C, B]] = \{B, \{C, D\}\} - \{C, \{B, D\}\} \; , \tag{A.9}$$

and the fact that the anticommutator of two traceless fields $B, C \in SU(2)$ can be written as a trace in flavor space

$$\{B, C\} = \langle BC \rangle \; . \tag{A.10}$$

Starting with the innermost (anti-) commutator (A_{n-1}, A_n) it is easy to see that the general polynomial A with n non singlet chiral fields decomposes to a linear combination of polynomials $\bar A$ of the form

$$\bar A = \begin{cases} \mathbf{1_d} \, \mathcal{O}_n \\ A_1 \, \mathcal{O}_{n-1} \\ [A_{i_1}, A_{i_2}] \; \mathcal{O}_{n-2} \end{cases} \tag{A.11}$$

where $\mathcal{O}_n$ stands for a generic product of three types of flavor traces

$$\begin{aligned} \mathcal{O}_n \; = & \langle A_1 \, [A_2, A_3] \rangle \dots \langle A_{j-2} \, [A_{j-1}, A_j] \rangle \\ & \times \langle A_{j+1} A_{j+2} \rangle \dots \langle A_{n-1} A_n \rangle \\ & \times S_1 \, S_2 \dots S_{n'} \end{aligned} \tag{A.12}$$

The operator A_i (S_i) stands for a non-singlet (singlet) chiral field and covariant derivatives thereof. For their definitions see (A.1) and (A.2).
Under charge conjugation such a polynomial transforms in a definite way

$$\bar A^c = (-1)^{c_{\bar A}} \bar A^T \, , \tag{A.13}$$

where

$$\begin{aligned} c_{\bar A} &= c_1 + \dots + c_n + n_v + n_{[\,]} \, , \\ n_{[\,]} &= \text{number of commutators in } \bar A \, , \\ n &= \text{number of non-singlet fields in } \bar A \, , \\ n_v &= \text{number of } v_{\mu\nu}^{(s)} \text{ in } \bar A \, , \end{aligned} \tag{A.14}$$

and c_k is the c-parity of the k'th chiral field A_k

$$A_k^c = (-1)^{c_k} A_k^T \, , \quad \text{(see Table 2)} \, . \tag{A.15}$$

This relation is valid because under charge conjugation the (anti-)commutator (A_1, A_2) transforms as

$$(A_1, A_2)_\mp = \mp (-1)^{c_1 + c_2} (A_1, A_2)_\mp \, . \tag{A.16}$$

In a similar fashion the hermiticity property of A can be analyzed

$$\begin{aligned} \bar A^\dagger &= (-1)^{h_{\bar A}} \bar A \, , \\ h_{\bar A} &= h_1 + \dots + h_n + n_{\chi_-} + n_{[\,]} \, , \\ n_{\chi_-} &= \text{number of } \langle \chi_- \rangle \text{ in } \bar A \, , \end{aligned} \tag{A.17}$$

where h_k is given by

$$A_k^\dagger = (-1)^{h_k} A_k \, , \quad \text{(see Table 2)} \, . \tag{A.18}$$

The parity of $\bar A$ is

$$\begin{aligned} \bar A^P &= (-1)^{p_{\bar A}} \bar A \\ p_{\bar A} &= p_1 + \dots + p_n + n_{\chi_-} + n_\epsilon \, , \\ n_\epsilon &= \text{number of } \epsilon^{\mu\nu\sigma\rho} \text{ in } \bar A \, . \end{aligned} \tag{A.19}$$

Here, p_k is the parity of the k'th chiral field A_k

$$A_k^P = (-1)^{p_k} \bar A \quad \text{(see Table 2)} \, . \tag{A.20}$$

It is understood that under parity lower Lorentz indices in A_k are contracted with the metric $g^{\mu\nu}$ and vice versa.

The most general term that can occur in $\mathcal{L}_{\pi N}$ without derivatives on the nucleon fields is then given by

$$\bar\Psi i^\epsilon \bar A \Psi \, , \quad \epsilon = 0, 1 \, . \tag{A.21}$$

The additional factor i is needed in the case when $\bar{A}$ is anti-hermitian. We are now in the position to derive the useful relations:

$$\left(\bar{\Psi}\Gamma i^{\epsilon}\bar{A}\Psi\right)^{c} = (-1)^{c_{\bar{A}}+c_{\Gamma}}\,\bar{\Psi}\Gamma i^{\epsilon}\bar{A}\Psi \tag{A.22}$$

$$\left(\bar{\Psi}\Gamma i^{\epsilon}\bar{A}\Psi\right)^{\dagger} = (-1)^{h_{\bar{A}}+h_{\Gamma}+\epsilon}\,\bar{\Psi}\Gamma i^{\epsilon}\bar{A}\Psi\,, \tag{A.23}$$

$$\left(\bar{\Psi}\Gamma i^{\epsilon}\bar{A}\Psi\right)^{P} = (-1)^{p_{\bar{A}}+p_{\Gamma}}\,\bar{\Psi}\Gamma i^{\epsilon}\bar{A}\Psi \tag{A.24}$$

with (see Table 3)

$$\left(\bar{\Psi}\Gamma\Psi\right)^{P} = (-1)^{p_{\Gamma}}\bar{\Psi}\Gamma\Psi\,, \quad C^{-1}\Gamma C = (-1)^{c_{\Gamma}}\Gamma^{T}\,,$$
$$\gamma_0\Gamma^{\dagger}\gamma_0 = (-1)^{h_{\Gamma}}\Gamma\,. \tag{A.25}$$

Thus an operator is permissible only if

$$(-1)^{p_{\bar{A}}+p_{\Gamma}} = (-1)^{c_{\bar{A}}+c_{\Gamma}} = (-1)^{h_{\bar{A}}+h_{\Gamma}+\epsilon} = 1\,. \tag{A.26}$$

In a next step we will allow for covariant derivatives acting also on the nucleon fields Ψ and $\bar{\Psi}$. Since the commutator between covariant derivatives can be written as

$$[\nabla_\mu, \nabla_\nu] = \frac{1}{4}\,[u_\mu, u_\nu] - \frac{i}{2}f^{+}_{\mu\nu} - iv^{s}_{\mu\nu} \tag{A.27}$$

any string of derivatives $i\nabla_{\mu_1}\ldots i\nabla_{\mu_n}$ acting on Ψ can be cast into the form

$$i\nabla_{\mu_1}\ldots i\nabla_{\mu_n}\Psi \to \left\{i\nabla_{\mu_1}, \{i\nabla_{\mu_2}, \ldots i\nabla_{\mu_n}\}\right\}\Psi\,, \tag{A.28}$$

up to terms with less than n derivatives acting on Ψ. The operators $\overrightarrow{D}^{n}_{\mu_1\ldots\mu_n}$ and $\overleftarrow{D}^{n}_{\mu_1\ldots\mu_n}$ are defined as

$$\overrightarrow{D}^{n}_{\mu_1\ldots\mu_n} = \{i\nabla_{\mu_1}, \{i\nabla_{\mu_2}, \ldots i\nabla_{\mu_n}\}\}\,, \tag{A.29}$$

$$\overleftarrow{D}^{n}_{\mu_1\ldots\mu_n} = \{i\overleftarrow{\nabla}_{\mu_1}, \{i\overleftarrow{\nabla}_{\mu_2}, \ldots i\overleftarrow{\nabla}_{\mu_n}\}\}\,, \tag{A.30}$$

where $\overleftarrow{\nabla}_\mu$ is given by

$$\overleftarrow{\nabla}_\mu = \overleftarrow{\partial}_\mu - \left(\Gamma_\mu - iv^{(s)}_{\mu}\right)\,. \tag{A.31}$$

Under charge conjugation, complex conjugation and parity they transform like

$$\left(\bar{\Psi}\,\overrightarrow{D}^{n}_{\mu_1\ldots\mu_n}\Psi\right)^{c} = \bar{\Psi}\overleftarrow{D}^{n}_{\mu_n\ldots\mu_1}\Psi\,, \tag{A.32}$$

$$\left(\bar{\Psi}\,\overrightarrow{D}^{n}_{\mu_1\ldots\mu_n}\Psi\right)^{\dagger} = (-1)^{h_D}\bar{\Psi}\overleftarrow{D}^{n}_{\mu_n\ldots\mu_1}\Psi\,, \quad h_D = n\,. \tag{A.33}$$

$$\left(\bar{\Psi}\,\overrightarrow{D}^{n}_{\mu_1\ldots\mu_n}\Psi\right)^{P} = \bar{\Psi}\,\overrightarrow{D}^{\mu_1\ldots\mu_n}_{n}\Psi\,. \tag{A.34}$$

The most general term in $\mathcal{L}_{\pi N}$ with n-derivatives on the nucleon can take one of the following forms:

a)

$$\bar{\Psi}\left(\overleftarrow{D}^{n}_{\mu_1\ldots\mu_n}\Gamma\bar{A} + (-1)^{h_D+h_{\bar{A}}+h_{\Gamma}}\bar{A}\Gamma\overrightarrow{D}^{n}_{\mu_n\ldots\mu_1}\right)\Psi \tag{A.35}$$

Charge conjugation invariance, parity and elimination of total derivatives require

$$(-1)^{p_{\bar{A}}+p_{\Gamma}} = (-1)^{h_D+c_{\bar{A}}+c_{\Gamma}} = (-1)^{h_{\bar{A}}+h_{\Gamma}} = 1 \tag{A.36}$$

b)

$$\bar{\Psi}\left(\overleftarrow{D}^{n}_{\mu_1\ldots\mu_n}\Gamma i\bar{A} - (-1)^{h_D+h_{\bar{A}}+h_{\Gamma}}i\bar{A}\Gamma\overrightarrow{D}^{n}_{\mu_n\ldots\mu_1}\right)\Psi \tag{A.37}$$

with

$$(-1)^{p_{\bar{A}}+p_{\Gamma}} = (-1)^{h_D+c_{\bar{A}}+c_{\Gamma}} = -(-1)^{h_{\bar{A}}+h_{\Gamma}} = 1 \tag{A.38}$$

Note the additional factor i in the term of type b) in (A.37). Lorentz invariance is again obtained by a suitable contraction of all Lorentz indices with $g^{\mu\nu}$ and $\epsilon^{\mu\nu\rho\sigma}$.

In the last step we will show that by a suitable redefinition of the nucleon field Ψ we can eliminate a certain class of operators of the form given in the two equations above. For this purpose we say that an operator of type a) or b) is of the order $\mathcal{O}(n, p^m)$, if n derivatives act on the nucleon fields and if the chiral order of $\bar{A}$ is $\bar{A} \backsim \mathcal{O}(p^m)$. Our strategy is the following: by a suitable redefinition of the nucleon fields we can substitute an operator of the order $\mathcal{O}(n, p^m)$ by operators $\mathcal{O}$ with less than n-derivatives on the nucleon field, i.e $\mathcal{O} \backsim \underset{j<n}{O}(j, p^m)$ and by operators with higher chiral order, $\mathcal{O} \backsim \mathcal{O}(n, p^{m+1})$. The successive redefinitions of the nucleon fields eliminates this operator up to terms with no derivatives on Ψ, which can be absorbed in the existing lagrangian $\mathcal{L}_{\pi N}$, and terms with higher chiral order.
Applying the nucleon field transformation

$$\Psi = \left[1 + \bar{A}\Gamma\overrightarrow{D}^{n-1}_{\mu_2\ldots\mu_n}\right]\Psi' \quad \bar{A} \backsim \mathcal{O}(p^m)\,,$$
$$\Gamma \in \text{Clifford Algebra} \tag{A.39}$$

to the leading order relativistic lagrangian $\mathcal{L}^{(1)}_{\pi N}$ in (2.15) generates the following lagrangian up to irrelevant forefactors

$$\begin{aligned}\mathcal{L}_{\text{ind}} &= \bar{\Psi}'\left[\overleftarrow{D}^{n}_{\mu_1\ldots\mu_n}\Gamma\gamma^{\mu_1}\bar{A}\right.\\ &\quad\left.+(-1)^{h_{\bar{A}}+h_{\Gamma}+h_D}\bar{A}\gamma^{\mu_1}\Gamma\overrightarrow{D}^{n}_{\mu_1\ldots\mu_n}\right]\Psi'\\ &\quad+\underset{j<n}{O}(j, p^m) + \underset{l\geqslant n}{O}(l, p^{m+1})\end{aligned} \tag{A.40}$$

The products $\Gamma\gamma^{\mu_1}$ and $\gamma^{\mu_1}\Gamma$ can be reduced to elements of the Clifford algebra and one obtains operators of the type a) or b).
As an example we choose $\Gamma = \gamma^\mu$. In this case the induced lagrangian is found to be

$$\begin{aligned}\mathcal{L}^{\gamma^\mu}_{\text{ind}} &= \bar{\Psi}'\left[\overleftarrow{D}^{n}_{\mu_1\ldots\mu_n}g^{\mu\mu_1}\bar{A}\right.\\ &\quad\left.+\,(-1)^{h_{\bar{A}}+h_D}\bar{A}g^{\mu\mu_1}\overrightarrow{D}^{n}_{\mu_1\ldots\mu_n}\right]\Psi'\\ &\quad+\bar{\Psi}'\left[\overleftarrow{D}^{n}_{\mu_1\ldots\mu_n}\Gamma\sigma^{\mu\mu_1}i\bar{A}\right.\\ &\quad\left.-(-1)^{h_{\bar{A}}+h_D}i\bar{A}\sigma^{\mu\mu_1}\overrightarrow{D}^{n}_{\mu_1\ldots\mu_n}\right]\Psi'.\end{aligned} \tag{A.41}$$

The first line is of type a) and the second is of type b). It is also possible to obtain the reverse situation by substituting $\Gamma \to i\Gamma$ in (A.39). Notice that there is a sign difference

between the two terms on each line. This means that one of these lines can be eliminated via partial integration up to operators with higher chiral order than $\bar{A}$. The remaining line can then be eliminated via the field redefinition (A.39). Letting Γ run through all elements of the Clifford Algebra one similarly finds

1.
$$\bar{\Psi}\bar{A}\,\Gamma^{\mu\mu_1}\overrightarrow{D}^n_{\mu_1\ldots\mu_n}\Psi + h.c. \backsimeq 0\,, \tag{A.42}$$
where $\Gamma^{\mu\mu_1} = \sigma^{\mu\mu_1}, \gamma_5\sigma^{\mu\mu_1}$

2.
$$\bar{\Psi}\bar{A}\,\Gamma\overrightarrow{D}^n_{\mu_1\ldots\mu_n}\Psi + h.c. \backsimeq 0\,, \tag{A.43}$$
where $\Gamma = \mathbf{1}, \gamma_5$

3.
$$\bar{\Psi}\bar{A}\,\Gamma^{\nu}\overrightarrow{D}^n_{\mu_1\mu_2\ldots\mu_n}\Psi \backsimeq \bar{\Psi}\bar{A}\,\Gamma^{\mu_1}\overrightarrow{D}^n_{\nu\mu_2\ldots\mu_n}\Psi, \tag{A.44}$$
where $\Gamma^{\mu} = \gamma^{\mu}, \gamma_5\gamma^{\mu}$

4.
$$\bar{\Psi}\bar{A}\,\epsilon^{\alpha\beta\mu\mu_1}\,\Gamma_{\mu}\overrightarrow{D}^n_{\mu_1\ldots\mu_n}\Psi + h.c. \backsimeq 0 \tag{A.45}$$

5.
$$\bar{\Psi}\bar{A}\,\epsilon^{\alpha\beta\mu\mu_1}\,\Gamma_{\mu\lambda}\overrightarrow{D}^n_{\mu_1\ldots\mu_n}\Psi + h.c. \backsimeq 0 \tag{A.46}$$

6.
$$\bar{\Psi}\bar{A}\,\Gamma^{\mu_1}\overrightarrow{D}^n_{\mu_1\ldots\mu_n}\Psi + h.c. \backsimeq 0\,. \tag{A.47}$$

Here "$\backsimeq$" stands for "equivalent to terms with less derivatives on Ψ and higher order terms". It should be noted, however, that our elimination procedure only works, if the chiral order of $\bar{A}$ is at least $\bar{A} \sim \mathcal{O}(p)$. In [35] the equations of motion have been used in order to remove six equation-of-motion type operators in the $SU(3)$ relativistic lagrangian. The first five operators are the $SU(3)$ versions of the respective terms in the above list. However, the last operator in [35] does not appear in our list.

Finally we indicate those terms with derivatives on the nucleon fields which cannot be eliminated by applying the rules given above. Let $\bar{A}^{\nu_1\ldots\nu_m}$ be a chiral operator of the form (A.11), where all Lorentz indices are fully contracted except for the indices $\nu_1\ldots\nu_m$. It is understood that no two of these indices are due to the same metric tensor $g^{\nu_i\nu_j}$ and that no two or more of the indices are due to the same antisymmetric tensor ϵ. The most general terms with n-derivatives on the nucleon field of the form (A.35) which cannot be eliminated by nucleon field redefinitions are then

$$\bar{\Psi}\left(\overleftarrow{D}^n_{\mu_1\ldots\mu_n}\Gamma_{\rho_1}\bar{A}^{\rho_1\mu_1\ldots\mu_n} + (-1)^{h_D+h_{\bar{A}}+h_\Gamma}\bar{A}^{\rho_1\mu_1\ldots\mu_n}\Gamma_{\rho_1}\overrightarrow{D}^n_{\mu_n\ldots\mu_1}\right)\Psi \tag{A.48}$$
with $\Gamma_{\rho_1} = \gamma_{\rho_1}, \gamma^5\gamma_{\rho_1}$ and

$$\bar{\Psi}\left(\overleftarrow{D}^n_{\mu_1\ldots\mu_n}\Gamma_{\rho_1\rho_2}\bar{A}^{\rho_1\rho_2\mu_1\ldots\mu_n} + (-1)^{h_D+h_{\bar{A}}+h_\Gamma}\bar{A}^{\rho_1\rho_2\mu_1\ldots\mu_n}\Gamma_{\rho_1\rho_2}\overrightarrow{D}^n_{\mu_n\ldots\mu_1}\right)\Psi \tag{A.49}$$

Table 2. P, C, and h.c. transformation properties of the chiral fields

	C	P	h.c.
u_μ	u_μ^T	$-u^\mu$	u_μ
$f^+_{\mu\nu}$	$-f^{+T}_{\mu\nu}$	$f^{\mu\nu}_+$	$f^+_{\mu\nu}$
$f^-_{\mu\nu}$	$f^{-T}_{\mu\nu}$	$-f^{\mu\nu}_-$	$f^-_{\mu\nu}$
$v^s_{\mu\nu}$	$-v^{sT}_{\mu\nu}$	$v^{\mu\nu}_s$	$v^s_{\mu\nu}$
$\chi+$	$\chi+^T$	$\chi+$	$\chi+$
$\chi-$	$\chi-^T$	$-\chi-$	$-\chi-$
$\overrightarrow{\nabla}_\mu$	$\overleftarrow{\nabla}^T_\mu$	$\overrightarrow{\nabla}^\mu$	$\overleftarrow{\nabla}_\mu$

Table 3. P, C, and h.c. transformation properties of the elements of the Clifford Algebra

	C	h.c		P
$\mathbf{1_d}$	$\mathbf{1_d}$	$\gamma^0\,\mathbf{1_d}\,\gamma^0 = \mathbf{1_d}$	$\bar{\Psi}\Psi$	$\bar{\Psi}\Psi$
γ^μ	$-\gamma^{\mu T}$	$\gamma^0\gamma^\mu\gamma^0 = \gamma^{\mu\dagger}$	$\bar{\Psi}\gamma_\mu\Psi$	$\bar{\Psi}\gamma^\mu\Psi$
γ^5	γ^{5T}	$\gamma^0\gamma^5\gamma^0 = -\gamma^{5\dagger}$	$\bar{\Psi}\gamma^5\Psi$	$-\bar{\Psi}\gamma^5\Psi$
$\gamma^5\gamma^\mu$	$\left(\gamma^5\gamma^\mu\right)^T$	$\gamma^0\gamma^5\gamma^\mu\gamma^0 = \left(\gamma^5\gamma^\mu\right)^\dagger$	$\bar{\Psi}\gamma^5\gamma_\mu\Psi$	$-\bar{\Psi}\gamma^5\gamma^\mu\Psi$
$\sigma^{\mu\nu}$	$-\left(\sigma^{\mu\nu}\right)^T$	$\gamma^0\,\sigma^{\mu\nu}\gamma^0 = \left(\sigma^{\mu\nu}\right)^\dagger$	$\bar{\Psi}\sigma_{\mu\nu}\Psi$	$\bar{\Psi}\sigma^{\mu\nu}\Psi$

with $\Gamma_{\rho_1\rho_2} = \sigma_{\rho_1\rho_2}, \gamma^5\sigma_{\rho_1\rho_2}$. An analogous expression holds for terms of the form (A.37). Consequently, if n derivatives act on the nucleon field the chiral order of $\bar{A}$ must be of the order $\bar{A} \sim \mathcal{O}(p^{n+1})$ for $\Gamma = \Gamma^{\rho_1}$ and of the order $\bar{A} \sim \mathcal{O}(p^{n+2})$ for $\Gamma = \Gamma^{\rho_1\rho_2}$. If one of the indices ν_j in $A^{\nu_1\ldots\nu_m}$ is due to the antisymmetric tensor ϵ the chiral order is further increased by at least one unit. These observations restrict severely the possible terms with n derivatives on the nucleon field of a given chiral order.

B Loop-Functions

We collect the loop integrals employed in this article. The following definitions and results have been used

$$\begin{aligned}\Delta(M_\pi^2) &= \frac{1}{i}\int\frac{d^dk}{(2\pi)^d}\frac{1}{M_\pi^2-k^2}\\ &= 2M_\pi^2\left[\Lambda(\mu) + \frac{1}{32\pi^2}\ln\frac{M_\pi^2}{\mu^2}\right],\end{aligned} \tag{B.1}$$

with

$$\Lambda(\mu) = \frac{\mu^{d-4}}{16\pi^2}\left\{\frac{1}{d-4} - \frac{1}{2}\left[\ln(4\pi) + \Gamma'(1) + 1\right]\right\}. \tag{B.2}$$

The integral $J_{\pi\pi}$ is defined as

$$J_{\pi\pi}(p^2) = \frac{1}{i}\int\frac{d^dk}{(2\pi)^d}\frac{1}{M_\pi^2-k^2}\frac{1}{M_\pi^2-(k-p)^2} \tag{B.3}$$

with explicit representation

$$J_{\pi\pi}(p^2) = \bar{J}_{\pi\pi}(p^2) + J_{\pi\pi}(0)$$
$$J_{\pi\pi}(0) = -2\Lambda(\mu) - \frac{2}{32\pi^2}\left[\ln\frac{M_\pi^2}{\mu^2}+1\right] + \mathcal{O}(d-4)$$
$$r = \left|1-4\frac{M_\pi^2}{p^2}\right|^{\frac{1}{2}}$$

$$\bar{J}_{\pi\pi}(p^2) = \begin{cases} \frac{1}{16\pi^2}\left[2-2r\arctan\frac{1}{r}\right], & 0<p^2<4M_\pi^2 \\ \frac{1}{16\pi^2}\left[2-r\ln\left|\frac{1+r}{1-r}\right|+i\pi r\right], & p^2>4M_\pi^2\,. \end{cases} \tag{B.4}$$

The integral K_0 is defined as

$$K_0(\omega,p^2) = \frac{1}{i}\int\frac{d^dk}{(2\pi)^d}\frac{1}{M_\pi^2-k^2}\frac{1}{M_\pi^2-(k-p)^2} \times\frac{1}{v\cdot k-\omega}\,, \tag{B.5}$$

and we find

$$K_0(0,p^2) = \begin{cases} \frac{1}{16\pi\sqrt{p^2}}\ln\frac{2M_\pi-\sqrt{p^2}}{2M_\pi+\sqrt{p^2}}\,, \\ \qquad \omega=0 \quad\text{and}\quad 0<p^2<4M_\pi^2 \\ \frac{1}{16\pi\sqrt{p^2}}\left[\ln\frac{\sqrt{p^2}-2M_\pi}{\sqrt{p^2}+2M_\pi}-i\pi\right], \\ \qquad \omega=0 \quad\text{and}\quad p^2>4M_\pi^2\,. \end{cases} \tag{B.6}$$

Finally, we have defined $J_0(\omega)$ as

$$\begin{aligned} J_0(\omega) &= \frac{1}{i}\int\frac{d^dk}{(2\pi)^d}\frac{1}{M_\pi^2-k^2}\frac{1}{v\cdot k-\omega} \\ &= -4\Lambda(\mu)\,\omega+\frac{\omega}{8\pi^2}\left[1-\ln\frac{M_\pi^2}{\mu^2}\right] \\ &\quad -\frac{1}{4\pi^2}\sqrt{M_\pi^2-\omega^2}\arccos\frac{-\omega}{M_\pi}\,. \end{aligned} \tag{B.7}$$

The derivative of $J_0(\omega)$ with respect to ω at $\omega=0$ is given by

$$\begin{aligned} J_0'(0) &= \frac{1}{i}\int\frac{d^dk}{(2\pi)^d}\frac{1}{M_\pi^2-k^2}\frac{1}{(v\cdot k)^2} \\ &= -4\Lambda(\mu)-\frac{1}{8\pi^2}\left[1+\ln\frac{M_\pi^2}{\mu^2}\right]. \end{aligned} \tag{B.8}$$

References

1. G.'t Hooft: Recent developments in gauge theories, eds. G.'t Hooft et al. (Plenum, NY, 1980); Y. Frishman, A. Schwimmer, T. Banks, S. Yankielowicz, Nucl. Phys. **B177**, 157 (1981); S. Coleman, B. Grossman, Nucl. Phys. **B203**, 205 (1982);
2. C. Vafa, E. Witten, Nucl. Phys. **B234**, 173 (1984); Comm. Math. Phys. **95**, 257 (1984)
3. S. Weinberg, Physica A **96**, 327 (1997)
4. J. Gasser, H. Leutwyler, Ann. Phys. **158**, 142 (1984); Nucl. Phys. **B250**, 465 (1984)
5. J. Stern, Two alternatives of spontaneous chiral symmetry breaking in QCD, preprint IPNO/TH 97-41 and hep-ph/9801282
6. J.L. Kneur, Phys. Rev. **D57,** 2785 (1998); G. Arvanitis, F. Geniet, J.L. Kneur, A. Neveu, Phys. Lett. **B390,** 385 (1997)
7. J. Stern, Light quark masses and condensates in QCD, Proc. of the Workshop on Chiral Dynamics (ChPT97), Mainz, Germany, Sept. 1-5, 1997, preprint hep-ph/9712438
8. N.H. Fuchs, H. Sazdjian, J. Stern Phys. Lett. **B269,** 183 (1991)
9. M. Knecht, J. Stern, Generalized Chiral Perturbation Theory, in *The Second DaΦne Physics Handbook*, L. Maiani, G. Pancheri, N. Paver (eds.), INFN-LNF (1995), preprint hep-ph/9411253
10. M. Knecht, E. de Rafael, Phys. Lett. **B424,** 335 (1998)
11. M. Knecht, B. Moussallam, J. Stern, N. Fuchs, Nucl. Phys. **B471**, 445 (1996); J. Stern hep-ph/9510318; M. Knecht, B. Moussallam, J. Stern, N. Fuchs, Nucl. Phys. **B457**, 513 (1995); M. Knecht, B. Moussallam, J. Stern, in *The Second DaΦne Physics Handbook*, L. Maiani, G. Pancheri, N. Paver (eds.), INFN-LNF (1995), preprint hep-ph/9411259; J. Bijnens et al., Phys. Lett. **B374,** 210 (1996); Nucl. Phys. **BB508**, 263 (1997); G. Ecker, Pion-Pion and Pion-Nucleon Interactions in Chiral Perturbation Theory, Proc. of the Workshop on Chiral Dynamics (ChPT97), Mainz, Germany, Sept. 1-5, 1997, preprint hep-ph/9710560
12. M. Baillargeon, P.J. Franzini, in *The Second DaΦne Physics Handbook*, L. Maiani, G. Pancheri, N. Paver (eds.), INFN-LNF (1995); B. Adeva et al., Lifetime measurement of $\pi^+\pi^-$ atoms to test low energy QCD predictions, CERN report, CERN/SPSLC 95-1
13. V. Bernard, N. Kaiser, U.-G. Meißner, Int.J.Mod.Phys. E **4**, 193 (1995)
14. N.H. Fuchs, H. Sazdjian, J. Stern, Phys. Lett. **B238,** 380 (1990)
15. S. Coleman, J. Wess, B. Zumino, Phys. Rev. **177,** 2239 (1969); C. Callan, S. Coleman, J. Wess, B. Zumino, Phys. Rev. **177,** 2247 (1969)
16. M. Knecht, private communication
17. G. Ecker, M. Mojžiš, Phys. Lett. **B410,** 266 (1997)
18. J. Gasser, M.E. Sainio, A. Švarc, Nucl. Phys. **B307**, 779 (1988)
19. T. Mannel, W. Roberts, Z. Ryzak, Nucl. Phys. **B368**, 204 (1992)
20. V. Bernard, N. Kaiser, J. Kambor, U.-G. Meißner, Nucl. Phys. **B388**, 315 (1992)
21. G. Ecker, Phys. Lett. **B336,** 508 (1994)
22. H. Georgi, Phys. Lett. **B240,** 447 (1990)
23. G. Ecker, M. Mojžiš, Phys. Lett. **B365,** 312 (1996)
24. J. Gasser, H. Leutwyler, M. E. Sainio, Phys. Lett. **B253,** 252 (1991)
25. L.S. Brown, W.J. Pardee, R.D. Peccei, Phys. Rev. **D4,** 2801 (1971)
26. V. Bernard, N. Kaiser, U.-G. Meißner, Phys. Lett. **B389,** 144 (1996)
27. J. Gasser, H. Leutwyler, M. E. Sainio, Phys. Lett. **B253,** 260 (1991)
28. M.E. Sainio, in Proc. of the Workshop on Chiral Dynamics: Theory and Experiment, Cambridge MA, USA, 25-29 July

1994, A.M. Bernstein, B.R. Holstein (Eds.), Springer, New York (1995)
29. M. Knecht, B. Moussallam, J. Stern, N. Fuchs, Nucl. Phys. **B471**, 445 (1996)
30. J. Kambor, R. Baur, in preparation
31. J.F. Donoghue, J. Gasser, H. Leutwyler, Nucl. Phys. **B343**, 341 (1990)
32. W.R. Frazer, J.R. Fulco, Phys. Rev. **117,** 1603 (1960)
33. G. Höhler, in Landolt-Börnstein, Vol. **9 b2**, ed. H. Schopper, Springer, Berlin (1983)
34. A. Schenk, Nucl. Phys. **B363**, 97 (1991)
35. A. Krause, Helvetica Phys. Acta **63**, 3 (1990)
36. N. Fettes, S. Steininger, U.-G. Meißner, Pion-nucleon scattering in chiral perturbation theory I: Isospin-symmetric case, preprint FZJ-IKP(TH)-1998-03 and hep-ph/9803266